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FOURTH CONFERENCE ON NONLINEAR VIBRATIONS, STABILITY, AND DYNAMICS OF STRUCTURES AND MECHANISMS

June 7-11, 1992

Abstracts

Virginia Polytechnic Institute and State University

Partially Sponsored by the Army Research Office

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The conference was held as scheduled. Abstracts of the presentations appear in a report distributed by the Virginia Polytechnic Institute and State University Sessions included identification and control, analytical methods, impact dynamics, dynamics and control, applications to cables, optimization and computational methods, coupled oscillators, applications to mechanical systems, random vibrations, analytical and symbolic methods, applications to structural elements, multibody dynamics, and flow-induced vibrations and computational methods.

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Sunday, June 7

1320-1330 **Opening Remarks**

Session 1. Identification and Control

Chairmen: L. N. Virgin, Duke University, Durham, NC and V. J. Modi, University of British Columbia, Vancouver, British Columbia, CANADA

1330-1510

Identification of Nonlinear Systems by using Hammerstein Feedback Models
S. Hanagud and J. Zhang, Georgia Institute of Technology, Atlanta, GA

Nonlinear System Identification without Model Form Assumptions
T. J. Meyer and D. J. Mook, The State University of New York at Buffalo, Buffalo, NY

Nonlinear Control of a Slewing Flexible Structure
R. W. Rietz and D. J. Inman, The State University of New York at Buffalo, Buffalo, NY

On the Optimal Control of Large Scale Mechanical Systems Subjected to Periodic Loading
P. Joseph, R. Pandiyan, and S. C. Sinha, Auburn University, Auburn, AL

A Heuristic Approach in the Optimal Control for Some Manipulators
B. Cheshankov, University of Sophia, Sophia, BULGARIA

1510-1530 **Break**

Session 2. Analytical Methods I

Chairmen: F. Pfeiffer, Technische Universitat Munchen, Munchen, GERMANY and A. C. Soudack, The University of British Columbia, Vancouver, British Columbia, CANADA

1530-1710

The Regional-Averaging Method and its Application to Chaotic Dynamics in Nonlinear Mathieu Systems
C. Yushu and Z. Weiyi, Tianjin University, Tianjin, CHINA

Existence and Non Existence of Periodical Solutions of the Liénard and Related Equations
S. Nocilla and P. Moroni, Politecnico di Torino, Torino, ITALY

The Loss of Stability of Periodic Attractors and their Transient Precursors
L. N. Virgin, P. V. Bayly, K. D. Murphy, J. A. Gottwald, and E. H. Dowell, Duke University, Durham, NC

Z2-Singularity Theory and 1/2 Subharmonic Bifurcation Responses of Mathieu-Duffing Equation
H. Guowei, Chinese Academy of Sciences, Beijing, CHINA and F. Tong North-Western Engineering University, Shaanxi, CHINA

Bifurcations in the Dynamics of an Orthogonal Double Pendulum
S. Samaranayake and A. K. Bajaj, Purdue University, West Lafayette, IN

1900-2100 **Reception**

Monday, June 8

Session 3. Impact Dynamics

Chairmen: A. K. Bajaj, Purdue University, West Lafayette, IN and A. Guran, University of Toronto, Toronto, Ontario, CANADA

0830-1010

Hammering in Diesel-Engine Driveline Systems
F. Pfeiffer, Technische Universitat Munchen, Munchen, GERMANY

Impact Phenomena of a Rotor-Casing Dynamical System
G. X. Li and M. P. Paidoussis, McGill University, Montreal, Quebec, CANADA

Investigation of the Dynamics and Bifurcations of an Impacting Spherical Pendulum with Large Deflections
S. Garza and A. Ertas, Texas Tech University, Lubbock, TX

Period-Infinity Periodic Motions, Chaos, and Sticking in a 10-Degree-of-Freedom Impact Oscillator
J. P. Cusumano and B. Bai, The Pennsylvania State University, University Park, PA

Continuous Contact Force Models for Impact Analysis in Multibody Systems
H. M. Lankarani, The Wichita State University, Wichita, KS and P. E. Nikravesh, University of Arizona, Tucson, AZ

1010-1030 **Break**

Session 4. Dynamics and Control

Chairmen: S. Hanagud, Georgia Institute of Technology, Atlanta, GA and D. J. Mook, The State University of New York at Buffalo, Buffalo, NY

1030-1210

A Singular Perturbation Analysis of the Equations of Servo-Hydraulics
R. Scheidl, Johannes Kepler University of Linz, Linz, AUSTRIA

Convergence and Asymptotic Stability of Hamiltonian Control Systems
A. J. Kurdila and F. Narcowich, Texas A&M University, College Station, TX

New Composite Adaptive Control Algorithm of Rigid Robots
A. A. Stotsky, Academy of Sciences of Russia, St. Petersburg, RUSSIA

Slewing Dynamics and Control of the Space Station Based Mobile Servicing System
V. J. Modi and A. Ng, University of British Columbia, Vancouver, British Columbia, CANADA

Effect of the Linearization of the Coriolis and Centrifugal Forces on the Feedforward Control Law of Flexible Mechanical Systems
M. Gofron and A. Shabana, University of Illinois at Chicago, Chicago, IL

1210-1330 **Lunch**

Session 5. Applications to Cables, Strings, and Rotors

Chairmen: S. W. Shaw, The University of Michigan, Ann Arbor, MI and F. Vestroni, University of L'Aquila, L'Aquila, ITALY

1330-1510

Two-to-One Internal Resonance in Suspended Elastic Cables
C. L. Lee and N. C. Perkins, The University of Michigan, Ann Arbor, MI

Transient Vibrations and Control of a Taut Inclined Cable with a Riding Accelerating Mass
I. Tadjbakhsh and Y. M. Wang, Rensselaer Polytechnic Institute, Troy, NY

Modal Interactions in a Parametrically and Externally Excited String
S. A. Nayfeh, A. H. Nayfeh, and D. T. Mook, Virginia Polytechnic Institute and State University, Blacksburg, VA

Behavior of a Cracked Rotating Shaft During Passage Through a Critical Speed
R. H. Andruet and R. H. Plaut, Virginia Polytechnic Institute and State University, Blacksburg, VA

Chaotic Motions and Fault Detection in a Cracked Rotor
P. C. Müller, J. Bajkowski, and D. Söffker, University of Wuppertal, Wuppertal, GERMANY

1510-1530 **Break**

Session 6. Optimization and Computational Methods

Chairmen: I. Tadjbakhsh, Rensselaer Polytechnic Institute, Troy, NY and I. I. Orabi, University of New Haven, West Haven, CT

1530-1710

An Efficient Algorithm for Elasto-Viscoplastic Vibrations of Multi-Layered Composite Beams Using Second-Order Theory
W. Brunner and H. Irschik, Johannes Kepler University of Linz, Linz, AUSTRIA

Singularity-Free Augmented Lagrangian Algorithms for Constrained Multibody Dynamics
E. Bayo, University of California, Santa Barbara, CA and A. Avello, University of Navarra and CEIT, San Sebastian, SPAIN

Constrained Optimization of Space Frame Structures
J. A. Czyz and S. A. Lukasiewicz, The University of Calgary, Calgary, Alberta, CANADA

Optimal Placement via Simulated Annealing of Passively Damped Struts in an Experimental 2-Dimensional Truss
T. A. Hamernik, The State University of New York at Buffalo, Buffalo, NY, E. Garcia, Vanderbilt University, Nashville, TN, and D. Stech, United States Air Force Academy, CO

Computation of Lyapunov-Floquet Transformation Matrices for General Periodic Systems
J. S. Bibb and S. C. Sinha, Auburn University, Auburn, AL

Tuesday, June 9

Session 7. Coupled Oscillators I

Chairmen: S. Wu, Air Force Office of Scientific Research, Bolling Air Force Base, DC and E. Bayo, University of California, Santa Barbara, CA

0830-1010

Forced Oscillations of a Rotating Shaft with Nonlinear Spring Characteristics and Internal Damping
Y. Ishida and T. Yamamoto, Nagoya University, Nagoya, JAPAN

Whirling of a Forced Cantilevered Beam with Static Deflection: Passage through Resonance
I-M. K. Shyu, Taipei, TAIWAN, D. T. Mook and R. H. Plaut, Virginia Polytechnic Institute and State University, Blacksburg, VA

Subharmonic Forced Traveling Waves in a Thin Perfect Circular Disk
T. A. Nayfeh and A. F. Vakakis, University of Illinois at Urbana-Champaign, Urbana, IL

Periodic Motion Through a Bifurcation
H. G. Davies and R. Pan, University of New Brunswick, Fredericton, New Brunswick, CANADA

Formulation of a Vibration Control Law Based on Internal Resonance
K. L. Tuer, M. F. Golnaraghi, and D. Wang, University of Waterloo, Waterloo, Ontario, CANADA

1010-1030 **Break**

Session 8. Applications to Mechanical Systems

Chairmen: D. J. Inman, The State University of New York at Buffalo, Buffalo, NY and S. Nocilla, Politecnico di Torino, Torino, ITALY

1030-1210

Chaotic Behavior of a Parametric Nonlinear Mechanical System
C. H. Lamarque and J. M. Malasoma, Ecole Nationales des Travaux Publics de L'Etat, Cedex, FRANCE

Chaos in the Unbalance Response of Journal Bearings
R. D. Brown, Heriot-Watt University, Edinburgh, SCOTLAND, P. S. Addison and A. H. C. Chan, Glasgow University, SCOTLAND

Nonlinear Response of a Class of Engine Mounts
A. G. Haddow, T. Onsay, and M. Brach, Michigan State University, East Lansing, MI

Dynamics of a Four Wheel Steer Vehicle
N. E. Sanchez, The University of Texas at San Antonio, San Antonio, TX

A Transmission Merit Parameter for Planar Mechanisms
F. Wu and H. M. Lankarani, The Wichita State University, Wichita, KS

1210-1330 **Lunch**

Session 9. Random Vibrations

Chairmen: R. A. Ibrahim, Wayne State University, Detroit, MI and H. Irschik, Johannes Kepler University of Linz, Linz, AUSTRIA

1330-1510

Applications in Nonlinear Soil/Structure Interaction
S. J. Serhan, Gilbert Commonwealth, Inc., Reading, PA

Chaotic Motion and Stochastic Excitation
F. Bontempi, Polytechnic of Milan, Milan, ITALY and F. Casciati, University of Pavia, Pavia, ITALY

Horizontal-Vertical Response Spectra for El Centro Earthquake
I. I. Orabi, University of New Haven, West Haven, CT and G. Ahmadi, Clarkson University, Potsdam, NY

Stochastic Response of a Parametrically Excited Buckled Beam to Wide-Band Random Excitation
A. M. Abou-Rayan and A. H. Nayfeh, Virginia Polytechnic Institute and State University, Blacksburg, VA

Lyapunov Exponents and Information Dimensions of Nonlinear Systems Under Deterministic and Stochastic Excitations
C. W. S. To and M. L. Liu, The University of Western Ontario, London, Ontario, CANADA

1510-1530 **Break**

Session 10. Analytical Methods II

Chairmen: A. Ertas, Texas Tech University, Lubbock, TX and S. A. Lukasiewicz, The University of Calgary, Calgary, Alberta, CANADA

1530-1710

The Nonstationary Period Doubling Route to Chaos
R. M. Evan-Iwanowski, University of Central Florida, Orlando, FL and C.-H. Lu, Memphis State University, Memphis, TN

Prediction of Escape from a Potential Well Under Harmonic Excitation
L. N. Virgin, Duke University, Durham, NC, R. H. Plaut and C. C. Cheng, Virginia Polytechnic Institute and State University, Blacksburg, VA

Assessing and Quantifying the Engineering Integrity of Nonlinear Vibrating Systems in Terms of Basins of Attraction
M. S. Soliman, University College London, London, ENGLAND

..ıfluence and Equivalence of Different Ship Roll-Damping Models through a Melnikov Analysis
M. Bikdash, B. Balachandran, and A. H. Nayfeh, Virginia Polytechnic Institute and State University, Blacksburg, VA

Nonlinear and Chaotic Oscillations of a Constrained Cantilevered Pipe Conveying Fluid: A Full Nonlinear Analysis
M. P. Paidoussis and C. Semler, McGill University, Montreal, Quebec, CANADA

1900 **Banquet**

Wednesday, June 10

Session 11. Analytical and Symbolic Methods

Chairmen: A. Vakakis, University of Illinois at Urbana-Champaign, Urbana, IL and J. Awrejcewicz, The University of Tokyo, Tokyo, JAPAN

0830-1010

On the Nonlinear Parametric Excitation Problems of a One and a Half Degrees of Freedom System
J. X. Xu, Xi'an Jiaotong University, Xi'an, Shaanxi Province, CHINA

The Symbolical Analysis of Nonlinear Systems of Differential Equations
G. S. Osipenko, State Technical University, St. Petersburg, RUSSIA

Nonlinear Vibration of a Flexible Connecting Rod
S. Hsieh and S. W. Shaw, The University of Michigan, Ann Arbor, MI

Chaotic Motion of a Gyrostat Satellite in a Circular Orbit
A. Guran, University of Toronto, Toronto, Ontario, CANADA

1010-1030 **Break**

Session 12. Applications to Structural Elements

Chairmen: S. C. Sinha, Auburn University, Auburn, AL and H. G. Davies, University of New Brunswick, Fredericton, New Brunswick, CANADA

1030-1210

On the Dynamic Behavior of a Flexible Beam Carrying a Moving Mass
F. Khalily, M. F. Golnaraghi, and G. R. Heppler, University of Waterloo, Waterloo, Ontario, CANADA

A Geometrically-Exact Beam Theory Accounting for Warpings and 3-D Stress Effects
P. F. Pai and A. H. Nayfeh, Virginia Polytechnic Institute and State University, Blacksburg, VA

Experiments on the Nonlinear Resonant Response of Thin Elastic Plates
S. A. McCabe, P. Davies, S. I. Chang, and A. K. Bajaj, Purdue University, West Lafayette, IN

Parametrically Excited Nonlinear Vibrations of Composite Flat Panels Exhibiting Initial Geometric Imperfections and Incorporating Non-Classical Effects
L. Librescu and S. Thangjitham, Virginia Polytechnic Institute and State University, Blacksburg, VA

Large Flexural Vibration of Thermally Stressed Layered Shallow Shells
R. Heuer, Technical University of Vienna, Vienna, AUSTRIA

1210-1330 **Lunch**

Session 13. Multibody Dynamics I

Chairmen: A. Shabana, University of Illinois at Chicago, Chicago, IL and J. X. Xu, Xi'an Jiaotong University, Xi'an, Shaanxi Province, CHINA

1330-1510

Experimental High Speed Response of a Slider Crank
D. Beale and D. Halbig, Auburn University, Auburn, AL

Steady State Response of a Slider Crank with Flexible Rod
D. Beale and S. Lee, Auburn University, Auburn, AL

The Inter-Relation Between Multibody Dynamics Computation and Nonlinear Vibration Theory
A. P. Kovacs and R. A. Ibrahim, Wayne State University, Detroit, MI

Steady-State Analysis of Large Scale Multibody Systems with Special Reference to Vehicle Dynamics
J. N. Lee and P. E. Nikravesh, The University of Arizona, Tucson, AZ

Damping of Parametrically-Excited SDOF Systems
K. A. Asfar, TH Darmstadt, GERMANY

1510-1530 **Break**

Session 14. Coupled Oscillators II

Chairmen: J. Wauer, Universitat Karlsruhe, Karlsruhe, GERMANY and Y. Ishida, Nagoya University, Nagoya, JAPAN

1530-1710

Iterated Maps in the Periodic Response of a Two DOF Elastoplastic System
D. Capecchi and F. Vestroni, University of L'Aquila, L'Aquila, ITALY

Analytical Construction of the Two-Parameter Family of Quasiperiodic Solutions in the Autonomous System
J. Awrejcewicz and T. Someya, The University of Tokyo, Tokyo, JAPAN

Constructing Invariant Tori for Two Weakly Coupled van der Pol Oscillators
D. E. Gilsinn, National Institute of Standards and Technology, Gaithersburg, MD

Normal Modes for Weakly Nonlinear Dynamical Systems
S. W. Shaw and C. Pierre, The University of Michigan, Ann Arbor, MI

Mode Localization in a System of Two Coupled Beams with Geometric Nonlinearities
M. E. King and A. F. Vakakis, University of Illinois at Urbana-Champaign, Urbana, IL

Thursday, June 11

Session 15. Multibody Dynamics II

Chairmen: G. Anderson, United States Army Research Office, Research Triangle Park, NC and P. E. Nikravesh, University of Arizona, Tucson, AZ

0830-1010

Multibody Dynamics of Aircraft Occupants Seated Behind Interior Walls
H. M. Lankarani, D. Ma, and R. Menon, The Wichita State University, Wichita, KS

Modelling of Vehicle Crash Tests by a Multibody System
J. P. Mizzi, Institut National de Recherche sur les Transports et leur Securite, Cedex, FRANCE

Intermittent Motion Analysis in Multibody Dynamics Using Joint Coordinates and Canonical Equations of Motion
M. S. Pereira, Technical University of Lisbon, Lisbon, PORTUGAL and P. E. Nikravesh, The University of Arizona, Tucson, AZ

On the Dynamics of Tethered Satellite Systems
J. Wauer, Universitat Karlsruhe, Karlsruhe, GERMANY

Nonlinear Motion of an Arbitrarily Shaped Satellite in an Elliptic Orbit Including the Effects of Damping
A. Guran, University of Toronto, Toronto, Ontario, CANADA and A. Vakakis, University of Illinois at Urbana-Champaign, Urbana, IL

1010-1030 **Break**

Session 16. Flow-Induced Vibrations and Computational Methods

Chairmen: R. M. Evan-Iwanowski, University of Central Florida, Orlando, FL and B. Cheshankov, University of Sophia, Sophia, BULGARIA

1030-1210

Nonlinear Dynamics of Articulated Cylinders Subject to Confined Axial Flow
M. P. Paidoussis and R. Botez, McGill University, Montreal, Quebec, CANADA

Weak and Strong Interactions in Vortex-Induced Resonant Vibrations of Cylindrical Structures
C. W. S. To, Q. S. Lu, and Z. S. Jin, The University of Western Ontario, London, Ontario, CANADA

An Efficient Numerical Technique for the Analysis of Parametrically Excited Nonlinear Systems
N. R. Senthilnathan and S. C. Sinha, Auburn University, Auburn, AL

A Mesh Repartitioning Scheme to Cope with Nonlinearities Resulting from Large and Fast Rotations of Deformable Bodies
A. Arabyan and T. Tsang, University of Arizona, Tucson, AZ

Response of Multi-Degree-of-Freedom Systems with Geometrical Nonlinearity Under Random Excitations by the Stochastic Central Difference Method
C. W. S. To, The University of Western Ontario, London, Ontario, CANADA

SESSION 1
IDENTIFICATION AND CONTROL
SUNDAY - 1320 - 1510
JUNE 7, 1992

Nonlinear System Identification Without Model Form Assumptions

Thomas J. Meyer and D. Joseph Mook
Department of Mechanical and Aerospace Engineering
State University of New York at Buffalo, Buffalo, NY 14260

Abstract

A robust nonlinear identification technique, based on the Minimum Model Error (MME) optimal estimation approach, is modified by a post-estimation correlation procedure to essentially eliminate any requirement of the user to assume the form of the nonlinear model. Model form is determined via statistical correlation of the MME optimal state estimates with the MME optimal model error estimates. The example illustrations, drawn from several physical systems, indicate that the method is robust with respect to prior ignorance of the model, and with respect to measurement noise, measurement frequency, and measurement record length.

The widespread existence of nonlinear behavior in many dynamic systems is well-documented. Many excellent methods for analyzing nonlinear system models have been developed. However, a key practical link is often overlooked: How does one obtain an accurate mathematical model for the dynamics of a particular complicated nonlinear system? The complexity of many real systems greatly diminishes the possibility of accurately constructing a dynamic model purely from analysis using the laws of physics.

Identification is the process of developing an accurate mathematical model for a system, given a set of output measurements and knowledge of the input. Many well developed and efficient identification algorithms already exist for linear systems. These often may be employed to model nonlinear systems when the system nonlinearities are small, and/or the system operates in a small linear regime. However, linearization does not work well (if at all) in every application, and even when it does provide a reasonable approximation, the approximation is normally limited to a small region about the operating point of linearization. Many important characteristics of nonlinear behavior, such as multiple steady-states, limit cycles, hysteresis, softening or hardening systems, chaos, etc., have no linear equivalent. Since nonlinearities are seldomly easily characterized, accurate nonlinear identification techniques are of high interest.

Numerous methods for the identification of nonlinear systems have been developed in the past two decades. In addition to the linearization approach, there are two other common approaches to nonlinear system identification: representing the system/nonlinearities using a series expansion, or assuming the form of the model a priori and then fitting parameters to the assumed model form. These existing techniques are subject to one or more of the following shortcomings: (i) For many techniques which require model form to be assumed a priori, the effort required to test a given form is considerable, which limits consideration of many different forms. (ii) Series expansions mask or eliminate understanding of the underlying physics. Moreover, many systems require a very large number of terms. (iii) The presence of noise in the measurement data is not rigorously treated, yet noise is generally unavoidable. (iv) Initial conditions must be known in order to implement the algorithm. (v) The algorithm can only be implemented if the data is obtained using very specific system excitations.

The technique of this paper is robust with respect to measurement noise; does not require knowledge of the initial conditions; is independent of the forcing; is not computationally prohibitive; and, most importantly, it requires minimal a priori assumptions regarding the form of the model or the system properties.

Nonlinear Control of a Slewing Flexible Structure

Ralph W. Rietz
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Mechanical & Aerospace Engineering
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Professor &Chairman
Mechanical & Aerospace Engineering
State University of New York at Buffalo
Amherst, New York 14260

Abstract

A nonlinear feedback control used to suppress vibrations and control the tip position of a slewing flexible structure is presented. The open loop system consists of a DC motor and a thin beam clamped at the hub (Figure 1). The motor and beam are modelled as an inertia-free system with an input torque at the hub. The open loop system is linear.

The controller is designed to control the rigid body mode and the first four flexible modes of the beam through a slewing maneuver. Using a multiple of state position and state velocity feedback provides a continuously variable damping term, which can improve the system response over optimal linear controllers. Several types of nonlinear feedback are presented. The simplest case involves sensing angular position and angular velocity at the hub. A linear PD controller can be designed and a multiple of the angular position and angular velocity yields the nonlinear damping. The second case requires the construction of a state estimator providing the states of the four flexible modes. The resulting feedback is equivalent to a full state feedback plus a nonlinear damping term, where the state positions and the corresponding state velocities are multiplied and fed back to the motor. In the third case, piezoceramics are used as sensors along the beam. The result is an improved feedback to the motor, since the flexible modes are more easily sensed along the beam. Also, using piezoceramics for collocated sensing and actuation is investigated. This leads to an interesting feedback situation, where a multiplication of position and velocity is fed back to the motor and a different, but similar, nonlinear term is returned to the piezoceramic for actuation. Any combination of the above mentioned controls is also possible. For example, angular position and angular velocity at the hub can be measured, a piezoceramic can be used near the hub on the beam to sense the relative motion of the beam to its rigid body state and a state estimator can be used to obtain any other states which are not directly sensed. Numerical simulations and experimental results of the various nonlinear feedback controls discussed above are presented. Figure 2 shows a simulation of the linear control system. Figure 3 shows the same system plus the nonlinear position times velocity feedback.

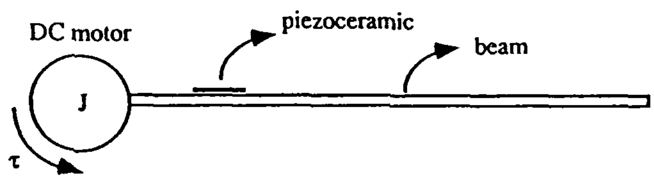

Figure 1: Motor and Beam Assembly

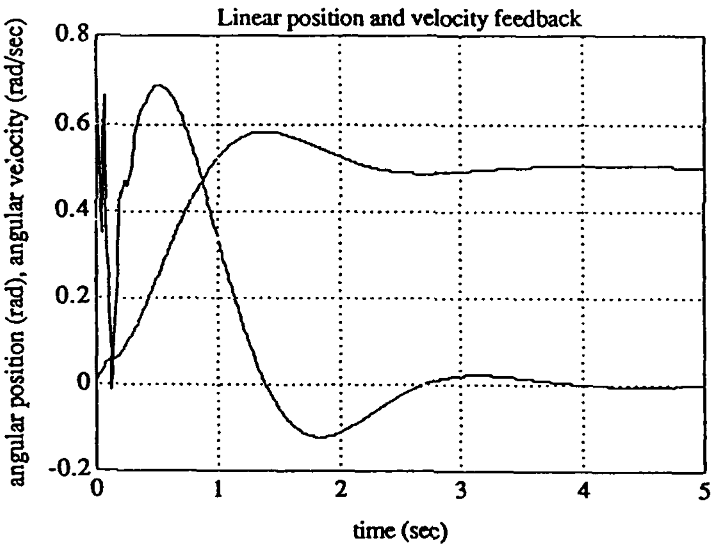

Figure 2: Response using linear control

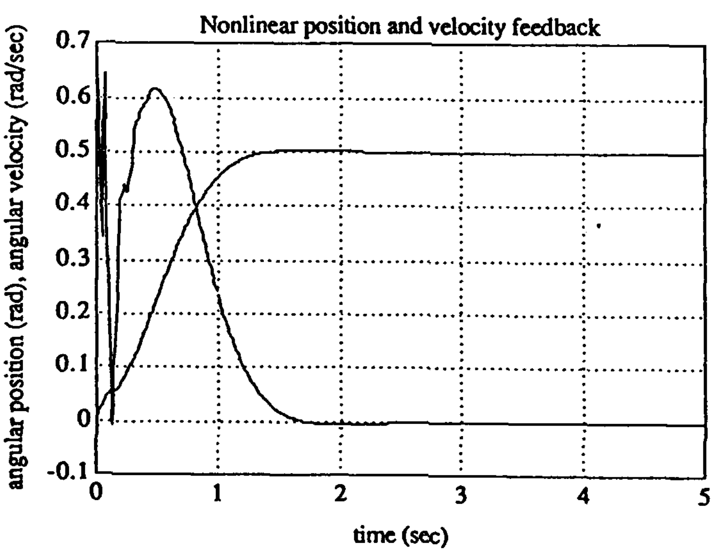

Figure 3: Response using nonlinear control

ON THE OPTIMAL CONTROL OF LARGE SCALE MECHANICAL SYSTEMS SUBJECTED TO PERIODIC LOADING

Paul Joseph, R. Pandiyan and S. C. Sinha
Department of Mechanical Engineering
Auburn University, Auburn, AL 36849

ABSTRACT

Mechanical systems subjected to periodic loading are modelled as a set of differential equations with periodic coefficients. For some time now, orthogonal polynomials have gained considerable attention in the stability and analysis of such systems. The major advantage of this procedure is in reducing the system equations to that of solving sets of algebraic equations. This algebraic method has been found more efficient than the standard numerical techniques[1], especially when the systems are large.

Often active control of structural characteristics is necessary to ensure desirable response of systems subjected to periodic loading. Control methodologies for general time-varying systems have been well reported[2]. In all, the optimal control strategy[3] stands out with distinct advantages. Also, it has been shown that the algebraic method and the optimal control strategy blends together well and provides a method for controller design of time-varying systems[4]. This paper presents an application of the above mentioned time-efficient algebraic method[1] to the control of large scale mechanical systems via optimal control theory. It is also shown that an observer based controller design for such systems can be achieved without resorting to the use of canonical transformations as reported in the literature[5].

The Control Problem: The control of a periodically varying system

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}(t)\boldsymbol{x}(t) + \boldsymbol{B}(t)\boldsymbol{u}(t)\,; \quad \boldsymbol{x}(0)=\boldsymbol{x}_0; \quad \boldsymbol{A}(t+T)=\boldsymbol{A}(t) \tag{1}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}(t)\boldsymbol{x}(t) \tag{2}$$

with respect to a quadratic performance index

$$\boldsymbol{J} = \frac{1}{2}\left(\boldsymbol{x}^T(t_f)\boldsymbol{S}(t_f)\boldsymbol{x}(t_f)\right) + \frac{1}{2}\int_0^{t_f}[\boldsymbol{x}^T(t)\boldsymbol{Q}(t)\boldsymbol{x}(t) + \boldsymbol{u}^T(t)\boldsymbol{R}(t)\boldsymbol{u}(t)]\,dt \tag{3}$$

is considered where all the vectors and matrices have usual properties as defined in reference[6].

As a first step, the 2n differential equations of the Linear Regulator problem in state and adjoint variables are formed. Then the state and adjoint vectors along with elements of the Hamiltonian matrix are expanded in terms of Chebyshev polynomials over the time period as elaborated in reference[1]. This reduces the original problem to a set of linear algebraic equations, the solutions of which provides the state transition matrix of the 2n system. Once this transition matrix is obtained, the controller gains are easily constructed as given in reference[6].

Observer Design for Periodic Systems: The form of the observer equation for the system (1) is given by

$$\dot{\boldsymbol{z}}(t) = \boldsymbol{F}(t)\boldsymbol{z}(t) + \boldsymbol{G}(t)\boldsymbol{y}(t) + \boldsymbol{H}(t)\boldsymbol{u}(t) \tag{4}$$

By selecting $\boldsymbol{F}(t) = \boldsymbol{A}(t) - \boldsymbol{G}(t)\boldsymbol{C}(t)$; $\boldsymbol{H}(t) = \boldsymbol{B}(t)$. and defining the error state as $\boldsymbol{e}(t) = \boldsymbol{x}(t) - \boldsymbol{z}(t)$ the error dynamics of the system is given by

$$\dot{\boldsymbol{e}}(t) = [\boldsymbol{A}(t) - \boldsymbol{G}(t)\boldsymbol{C}(t)]\boldsymbol{e}(t) \tag{5}$$

The $\boldsymbol{G}(t)$ is computed such that error dynamics is asymptotically zero as the time goes to infinity. The transpose of the gain matrix of the dual system of (1) provides $\boldsymbol{G}(t)$ and the computation procedure is as described above.

Numerical Results and Conclusions: As an example of large scale mechanical system, consider the inverted triple pendulum problem bearing a periodic directional load as shown in figure 1. The problem has been solved for single, double and triple inverted pendulum models in order to show the viability of the technique for large scale systems. As a typical example, results of open loop behavior of the double inverted pendulum under the periodic loading is provided in figure 2. Then the control systems were designed using both direct feedback and observer based feedback laws and the corresponding closed loop response of the double inverted pendulum are depicted in figures 3 and 4 respectively. It has been found that the results obtained using this technique, matches with the numerical integration results for relatively small number of Chebyshev coefficients. Many attractive features of the present technique are noted namely, (i) reduction of the control problem to a process of solving a set of algebraic equations, (ii) the computational efficiency compared to numerical procedures as the system becomes larger and larger, (iii) the adaptability to parallel processing machines and (iv) finally, ability to place poles of closed loop time-varying systems through optimal control theory.

Acknowledgements: Financial support from ARO contract no. DAAL03-89-k-0172 monitored by Dr. Gary L. Anderson is gratefully acknowledged.

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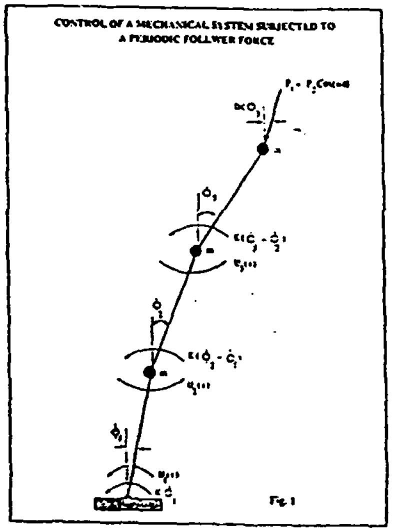


Fig.2 UNCONTROLLED STATES OF THE DOUBLE PENDULUM

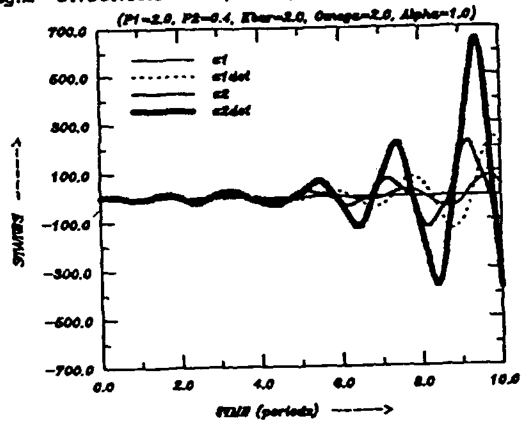


Fig.3 FULL-STATE FEEDBACK CONTROL OF THE DOUBLE PENDULUM

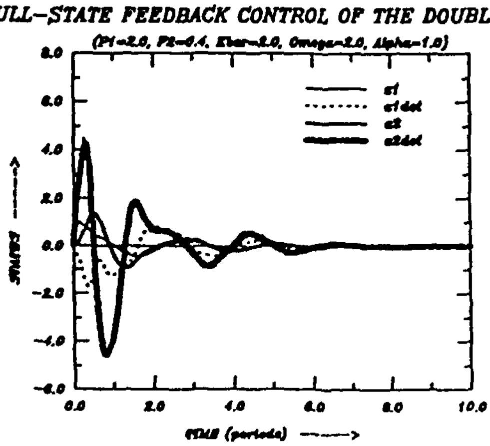
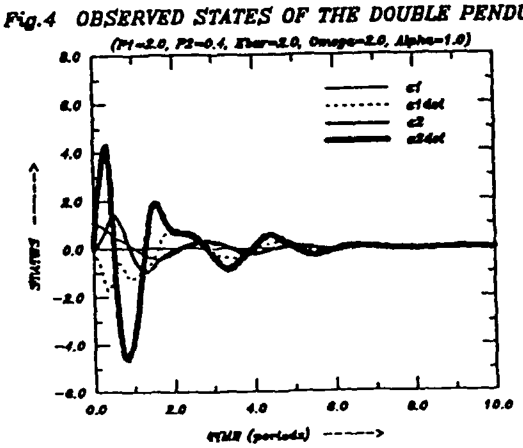


Fig.4 OBSERVED STATES OF THE DOUBLE PENDULUM



A HEURISTIC APPROACH IN THE OPTIMAL CONTROL FOR SOME MANIPULATORS

B. Cheshankov

A motion of a manipulator with three degrees of freedom is considered. One of the coordinates corresponds to a rotation about a vertical axis. When one specifies the two other coordinates a manipulator which operates in cylindrical coordinate system or a manipulator which operates in spherical coordinate system is described. The motion of an anthropomorphous manipulator can be described too. The problem of time-optimal control is stated. To seek such control which satisfies some constraints and guides the gripper of the manipulator from an initial state to a given finite state within a minimal time. The main point of the heuristic approach is to guide the manipulator from the initial state to a state which corresponds to a configuration of the manipulator in which the moment of inertia of the whole system about the axis of rotation reaches its minimal value (while the coordinate which describes the rotation increases monotonously). During some time a motion with maximal acceleration (because of the minimal value of the moment of inertia) is executed. After this the manipulator should be guided to the finite state. Various cases are possible which depends on the initial conditions. For some initial conditions the minimal value of the moment of inertia could not be reached (there is not enough time for this) and this process should be stopped at some point. After this the manipulator should be guided to the finite state. In all cases the finite values for all coordinates should be reached simultaneously. The singularities for the motion of different concrete manipulators when employing this heuristic approach are discussed. Some numerical examples are given.

SESSION 2
ANALYTICAL METHODS I
SUNDAY - 1530 - 1710
JUNE 7, 1992

The Regional–Averaging Method and its Application to Chaotic Dynamics in Nonlinear Mathieu Systems

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Because of the simple calculation for averaging method,a great deal of local bifurcation problems were solved by averaging method or improved averaging method by many people ,such as,Chow[1],Bryno[2],Van Der Beek [3] , Chen Yushu [4] ,etc.In these works,people calculated only the first order approximate solution. Can we express the first order approximate solution of nonlinear systems by more simple analysis? It needs to study the case of existing strong nonlinear terms in its derivative systems for studying the global problem, in this time, the classical averaging method is not suitable. There are some averaging methods for the study of stable response,such as,the generalized averaging method , the method of expanding in ellipse function and Melnikov method. There are a great difference among the forms of the results by different methods. It is hard to tell whether they are equivalent. Comparablely, Melnikov method is the unique systematically method in analysing global bifurcation and chaos , but it can give only the necessary condition of existing subharmonic solution and chaos solution. For the complicated systems, especially for the systems with multi–frequencys forcing ,it is difficult for using Melnikov method to analyse.In fact,the critical value of existing subharmonic solution and chaos solution by Melnikov method is a approximate range, it is lower limit. It makes numerical or experimental examining difficult and impossible. So, it is very important to study the sufficient and necessary condition of existing subharmonic solution and chaos solution.

In this paper, we present a new method, Regional– Averaging Method (RA method)[5] . RA method is different from existing averaging methods. It does not need to transform the equation, (Such as, coordinate transformation, KB transformation, etc) and can get the approximate solution directly. The local bifurcation, global bifurcation and chaos can be analysed by RA method. In the study of local bifurcation,the results by RA method are equivalent to the first order approximate results by existing averaging methods. In the study of global bifurcation, RA method can analyse the the sufficient and necessary condition of existing subhamonic solution and chaos solution. Comparing with Melnikov method, we point out that in the case of nonresonance, Melnikov method gives the sufficient and necessary condition. in the case of resonance, Melnikov method gives only the necessary condition. RA method can give the sufficient and necessary condition.

Existence and non existence of periodical solutions of the Liénard and related equations

by Silvio Nocilla and Paola Moroni

In a recent paper [1] sufficient conditions are obtained in order that the Liénard Equation:

$$\ddot{x} + f(x)\,\dot{x} + g(x) = 0 \tag{1}$$

don't have periodical solutions, thous limit cycles don't exist. It is well known, see for inst.[2], Chap.3 , that a large lot of papers are devoted to the approximated calculation of such limit cycles, in particular for the Van der Pol or Rayleigh Equation. The present paper, which can be related to a previous paper of the first Author [3], deals with following topics:

a) <u>to give a procedure for exact calculation of the limit cycles.</u> The procedure is the same applied in [3] and in other papers, for instance [4] and [5]. Namely we look for a periodical monoscillating solution in the parametric form:

$$\begin{cases} x = x^* \sin\tau & \tau \in [-\pi/2, \pi/2] \\ \Omega t = \tau + \int_{-\pi/2}^{\tau} \varphi(s)\,ds \end{cases} \tag{2}$$

and we obtain a fixed-point problem in a Banach space for the unknown form function $\varphi(\tau)$, with auxiliary conditions, also involving the unknown constants (x^*, Ω). Particularly interesting is the fact that no-assumptions on "small non-linearity parameters" are introduced.

b) <u>to give a criterion for the existence of periodical-monoscillating solutions</u>, and to test this criterion in the particular cases. For Eq.(1) <u>the criterion is that setting:</u>

$$\begin{cases} F(x) = \int_0^x f(\xi)\,d\xi \\ H(f,g;\tau;x^*,\Omega) = \Omega[F(x^*\sin\tau) - F(-x^*)] + \int_{-\pi/2}^{\tau} g(x^*\sin s)\,ds \end{cases} \tag{3}$$

the system of algebric Eqs:

$$\begin{cases} H(f,g;\pi/2;x^*,\Omega) = 0 & (4) \\ \Omega^2 x^* + \int_{-\pi/2}^{\pi/2} \dfrac{H(f,g;\tau;x^*,\Omega)}{\cos\tau}\,d\tau = 0 & (5) \end{cases}$$

<u>has a real solution</u> $(x^*, \Omega) \in R^2_+$.
The definite integral in (5) is <u>convergent</u> in force of conditon (4).
The criterion can be easy generalized to more general Equations.

c) <u>to show that also for other classes</u> of Liénard's equations not satisfyng to the conditions given in [1], the non-existence property is true. For instance the Eq:

$$(6) \qquad \ddot{x} - \mu(1+x^2)\,\dot{x} + x = 0$$

don't has periodical solutions. Another example is given by Eq:

$$(7) \qquad \ddot{x} - \mu(1-x^2)\,\dot{x} + x + k\,x^3 = 0$$

for which, according to the above criterion, the periodical solution exist only for $k>-2/5$. A third example is given by Eq:

$$(8) \qquad \ddot{x} - \mu(1-x^2)\,\mathrm{sgn}\,\dot{x} + x = 0$$

already considered in [2], for which periodical solutions exist only for $\mu \leq \sqrt{3}/2$.

For Equation:

$$(9) \qquad \ddot{x} + (10+x)\,\dot{x} + x = 0$$

considered in [1] as 1^{st} example, the above criterion leads to two conditions: the 1^{st} one is:

$$20\,\Omega\, x^* = 0$$

which clearly has no solutions $(x^*, \Omega) \in R_+^2$; the second one is meaningless because it contains a divergent integral. Thous our criterion agrees with the conclusion drawn in [1].

d) <u>to carry out the exact calculation</u> of the limit cycle with iterative procedure when the conditions of the criterion are satisfied, and to study the character of the solutions (my be chaotic solutions) when the conditions don't hold.

The results obtained with the parametric method are compared with standard Runge-Kutta numerical integration.

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The loss of stability of periodic attractors and their transient precursors

by

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Abstract

The behavior of a harmonically excited, nonlinear mechanical oscillator is dominated by the presence of periodic attractors which compete to capture local transients. Each of these stable, steady-state solutions has associated with it a catchment region of initial conditions. These catchment regions are delineated by separatrices which pass through the unstable solutions. Under the smooth variation of certain system parameters, such as forcing frequency or damping, catchment regions may erode and bifurcations may occur, signalling a qualitative change in behavior. In practical terms a stable oscillation may lose its stability.

The loss of stability of a periodic attractor is reflected in the rate of attraction of transients. This feature underlies classical Floquet theory. These transients may be caused by start-up conditions, for example exciting a system initially at rest, or may be caused by inevitable perturbations to a steady-state. It is this latter kind of transient which is induced in the current study. The ensuing motion and its rate of attraction is then monitored as a function of the changing control parameter and stability predictions are made.

For a single-degree-of-freedom system under the operation of one control parameter, two common generic mechanisms of instability are the saddle-node bifurcation associated with the jump phenomenon [1] and the flip bifurcation associated with the initiation of a sequence of period doubling. It is well established that both of these bifurcations are related to the penetration of a characteristic multiplier through the unit circle.

Suppose a system is exhibiting stable steady-state behavior. Now if a small perturbation is induced, the characteristic multipliers govern the behavior of the subsequent transient as it decays onto the periodic attractor. Suppose that the control parameter is varied by means of a quasi-static increment or a slowly changing function of time. Poincare mapping techniques can be employed to construct a local (linear) description of the transient behavior and, from the subsequent approximation to the Jacobian matrix, characteristic multipliers can be obtained. It is the evolution of these characteristic multipliers that reflects the approach to instability.

This may be considered as the numerical analogy of Floquet theory, and in the current paper this technique is applied to both numerical simulations and mechanical experiments. Two archetypal nonlinear mechanical models are chosen to illustrate the technique: a geometrically nonlinear spring-mass-damper and its approach to the jump phenomenon, and an impacting pendulum and its approach to period doubling [2]. It is shown that incipient instability can be predicted using transient dynamical effects. Extensions to the current method hold considerable promise in the non-destructive monitoring of deteriorating dynamical systems.

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The Paper Submitted to Fourth Conference on Nonlinear Dynamics

Z_2-Singularity Theory and 1/2 Subharmonic Bifurcation Responses of Mathieu-Duffing Equation

(Abstract)

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Since the Hopf bifurcation diagrams in the autonomous systems were obtained with singularity theory, the attempt to obtain the subharmonic bifurcation diagrams in the periodic parametric excited systems has been made by use of averaging equations and amplitude equations, whose bifurcation diagrams can be investigated by the singularity theory of one variable. however, the averaging equations or amplitude equations are not contact equivalent with the parametric excited system. it is following that their bifurcation diagrams are different topologically from the bifurcation diagrams of the parametric excited systems. In this paper the 1/2-subharmonic bifurcation problems of the parametric excited systems (e.g. Mathieu-Duffing equation) are investigated under the Z_2-contact equivalence in order to obtain the bifurcation diagrams which are topological equivalent to the subharmonic bifurcation diagrams of the periodic parametric excited systems.

Firstly the 1/2 subharmonic bifurcation problems of the Mathieu-Duffing equation are transformed to the complex algebraic bifurcation equation via the Liapunov-Schmidt reduction. The complex algebraic bifurcation equation, which is leading equivalently to two real variable (not one !) algebraic bifurcation equations, are Z_2-contact equivalent to the Mathieu-Duffing equation. Therefore, we must use the Z_2-equivarient singularity of two real variables to investigate their bifurcation diagrams. Secondly the Z_2-equivarient singularity theory of two variables is formulated, including the recognition theorem and universal unfolding theory. Finally the overall 1/2 subharmonic bifurcation diagrams of the Mathieu-Duffing equation are obtained in the polar coordinates with the above theorem. The numerical simulation is agreement with our theoretic results.

The Z_2-equivarient singularity theory of two variables and corresponding judgment conditions are not only suitable to the study on 1/2 subharmonic responses of the Mathieu-Duffing equation, but also suitable to the other periodic parametric excited systems. And more the Z_n-equivarient singularity theory is also formulated in the author's doctorical thesis, which can be used to study 1/n subharmonic bifurcation diagrams of the periodic parametric excited systems.

BIFURCATIONS IN THE DYNAMICS OF AN ORTHOGONAL DOUBLE PENDULUM

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The weakly nonlinear resonant response of an orthogonal double pendulum to planar harmonic motions of the point of suspension is investigated. The two pendulums in the double pendulum are confined to two orthogonal planes. For nearly equal length of the two pendulums, the system exhibits 1:1 internal resonance. The method of averaging is used to derive a set of four first order autonomous differential equations in the amplitude and phase variables. Constant solutions of the amplitude and phase equations are studied as a function of physical parameters of interest using the local bifurcation theory. It is shown that, for excitation restricted in either plane, there may be as many as six pitchfork bifurcation points at which the nonplanar solutions bifurcate from the planar solutions. These nonplanar motions can become unstable by a saddle-node or a Hopf bifurcation, giving rise to a new branch of constant solutions or limit cycle solutions, respectively. The dynamics of the amplitude equations in parameter regions of the Hopf bifurcations is then explored using direct numerical integration. The results indicate a complicated amplitude dynamics including multiple limit cycle solutions, period-doubling route to chaos, and sudden disappearance of chaotic attractors.

SESSION 3
IMPACT DYNAMICS
MONDAY - 0830 - 1010

JUNE 8, 1992

HAMMERING IN DIESEL-ENGINE DRIVELINE SYSTEMS

Friedrich Pfeiffer, München

Machines and mechanisms are characterized by rigid or elastic bodies interconnected in such a way that certain functions of the machines can be realized. Couplings in machines are never ideal but may have backlashes or some properties which lead to stick-slip phenomena. Under certain circumstances backlashes generate a dynamical load problem if the corresponding couplings are exposed to loads with a time-variant character. A typical example can be found in gear systems of diesel-engines, which usually must be designed with large backlashes due to the operating temperature range of such engines, and which are highly loaded with the oscillating torques of the injection pump shafts and of the camshafts. Therefore, the transmission of power from the crankshaft to the camshaft and the injection pump shaft takes place discontinuously by an impulsive hammering process in all transmission elements.

Mechanically, processes of that type are unsteady vibrations characterized by a time series of impulsive events and thus belonging to the class of nonlinear vibrations with an unsteady behaviour. As a rule such vibrations may be periodic, quasiperiodic or chaotic with a tendency to chaos for large systems. Considering the driveline-gear-system as a multibody system with totally f degrees of freedom and with n_p backlashes in the gear meshes or in the bearings we model the backlash properties by a nonlinear force characteric with small dissipative forces within the backlash and a linear force law in the case of contact of the flanks. The event of a contact is determined by an evaluation of the relative distance in each backlash, which serves as an *indicator function*. The indicator function for leaving the contact, i.e. flank separation, is given with the normal force in the point of contact, which changes sign in the case of flank separation. These unsteady points (switching points) must be evaluated very carefully to achieve reproducable results. The time series of impact forces will be reduced to load distributions in a last step. They might serve as a basis for life time estimates. It turns out that distributions in the form of Gamma-functions and related relations represent the unsteady hammering process surprisingly well.

Theory has been compared with measurements at a 4-stroke 12 cylinder diesel-engine with 3000, where experiments have been performed by a German diesel-engine manufacturer. Simulations and experiments agree quite well.

Impact Phenomena of a Rotor-Casing Dynamical System

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Rub between a rotor and its supporting device is a serious malfunction in rotating machinery. In this talk, a shaver rotor-casing system with clearance is modeled by two second-order differential equations with zero stiffness, and the dynamics is investigated through analytical form of solutions as well as numerical simulation. The results demonstrated the existence of whirling behaviour (rubbing), periodic and quasi-periodic impacts, and in some cases chaotic solutions. A Lyapunov exponent technique is developed to characterize the system behaviour due to changes of system parameters.

In nondimensional form, the system governing equations can be written as follows:

$$\ddot{x} = e\cos(t), \qquad \ddot{y} = e\sin(t), \tag{1}$$

where x and y represent the position of the geometric centre of the rotor, and e is the eccentricity of the unbalance. Upon impacting, the rotating rotor either continuously rubs against the casing in the same direction as the rotor speed (forward whirling) or in an opposite direction (backward whirling), or it may bounce back and forth within the clearance. If the normal and tangential velocity components, before impacting, are denoted by v^n_- and v^t_-, and those after impacting by v^n_+ and v^t_+, then the impact rules between the rotor and the casing may be expressed as

$$v_+^n = -\alpha v_-^n, \qquad v_+^n = (1-B)\, v_-^t, \tag{2}$$

where α is the coefficient of restitution, representing the energy loss caused by impacting, and B is the breaking coefficient and is given by

$$B = \mu(1+\alpha)\frac{v_-^n}{v_-^t}. \tag{3}$$

Such kind of rotating systems has been studied extensively in the past with a variety of models, but most of the work were dealing with the whirling behaviour of the system. Ehrich [1] and Billet [2] considered only the case of backward whirling and neglected any external forcing such as that due to mass unbalance. They basically came to the same conclusion as Den Hartog [3] that if the rotor runs under normal operation condition it is stable due to spring restoring forces; but as soon as it is perturbed from its central position and temporarily deflected so as to striking the casing, it is set into backward whirling. Johnson [4], on the other hand, considered the eccentricity of the rotor but neglected the effect of friction on the system response. He concluded that upon contact, only steady-state synchronous whirl could be possible. In the proposed model here, both whirling behaviours were observed. Furthermore, through numerical simulation, regions were also observed in parameter space where no whirling is possible but the rotor starts impacting with its casing. Two such impact behaviours are shown in Figures 1 and 2 in the (x,y) plane. In Figure 1 the rotor impacts with the casing in the same direction as its rotating speed and the response is quasi-periodic. For some larger values of the dry friction and eccentricity, the resulting trajectory has become highly irregular, as shown in Figure 2.

In the impact regions, analytical solutions may be obtained for equation (1) in the time interval between any two consecutive impacts. Thus the system (1), together with the impact rule (2), may be converted into a set of discrete equations. Consequently, the Lyapunov exponent technique can be applied to the discretized equations to characterize the system dynamic behaviour. Numerical calculations of Lyapunov exponents show that the response in Figure 1 is indeed quasi-periodic with a zero exponent, and the one in Figure 2 is chaotic resulting in a positive exponent.

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Fig.1: e = 0.05, μ = 0.014

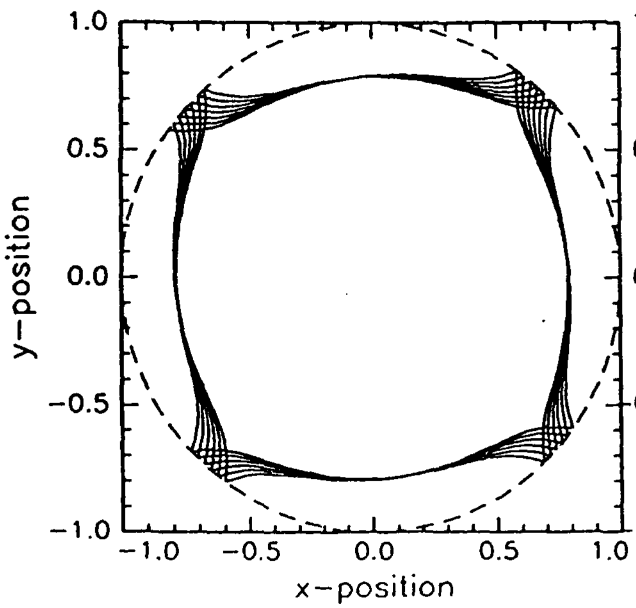
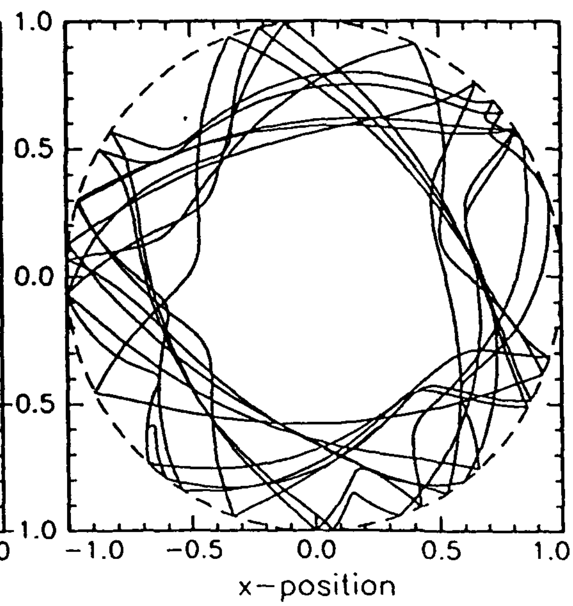


Fig.2: e = 0.1, μ = 0.023



Investigation of the Dynamics and Bifurcations of an Impacting Spherical Pendulum with Large Deflections

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ABSTRACT

The inverted spherical pendulum is a common engineering paradigm for strong focusing mechanisms. Strong focusing mechanisms have practical application in laser stability, synchatrons, magneto plasma confinement, long distance laser focusing and containment of charged particles. The system has direct impact on damping nutation of space craft and vibration absorbtion in helicopters. A model of such a pendulum system with vertical forcing, large deflection and impacting is being studied. The model will include quadratic and Coulomb damping. It will be shown that the two coupled equations of motion for the system reduce to one in the presence of damping for long term motion. An autonomous exact solution of the parametrically forced pendulum model with quadratic and Coulomb damping is studied. This solution of the equation of motion would allow solutions for minimum conditions of wall impact, type I, type II motions and Melnikov bifurcation analysis. Vector plots of the solution for varying initial conditions of position and velocity will be shown. Theoretical analysis shows that the inversion criteria for a non-impacting parametrically forced spherical pendulum is similar to that of a parametrically forced plane pendulum. Analysis also suggests the interesting point that the system will exhibit type II motion (striking alternate walls) at lower energy input levels than are required for type I motions (striking the same wall repeatedly). Melnikov analysis will be performed on the system at the initial condition of $\theta = \pi$ for varying initial angular velocities.

Period-Infinity Periodic Motions, Chaos, and Sticking in a 10-Degree-of-Freedom Impact Oscillator

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Abstract

In this paper, the numerical study of a 10-degree-of-freedom impact oscillator is presented. The mathematical model is of a lumped mass system in which each mass is connected to its nearest neighbors by linear springs and linear viscous dampers. One end mass is connected to a rigid support, while the other is free to impact with a sinusoidally moving rigid table. Bifurcation diagrams based on the impact map are obtained over the entire range of natural frequencies of the system. The diagrams (see, e.g., Fig. 1) reveal many chaotic bands, as well as a wide variety of period-n, m orbits, where n is the period with respect to the impact map and m is the period with respect to the table motion. The type of period-n, m orbits and the existence of chaos is ascertained by examining projections of the Poincaré sections (as in Fig. 2). Chaos is also confirmed by direct study of sensitive dependence on initial conditions. Perhaps the most interesting solutions are period-∞, m periodic orbits (where m is finite). These arise from *sticking events* where the time between impacts approaches zero in finite time.

Since all of the nonlinearity in the problem is contained in the impact boundary condition, analytical solutions are easily found for the system motion between impacts. These solutions are then used to construct the implicit 20-dimensional Poincaré map defined by the impact condition. The only variable that changes across the impact is the velocity of the tip mass which collides with the table: its value after impact is calculated using the elementary impact law

$$v_{10}^{+} = e\,(v_T - v_{10}^{-}) + v_T \,,$$

where e is the coefficient of restitution, v_{10} and v_T represent the velocity of the end mass and moving table, respectively, and the instants just before and just after impact are represented by the symbols "$-$" and "$+$", respectively. A numerical code based on Newton's method is used to find the time Δt_i between impact i and $i+1$.

A naive implementation of the impact Poincaré map algorithm is very efficient for orbits with average impact time intervals which are not too small. However, in chaotic orbits, and certain periodic orbits, sticking events can occur in which Δt_i rapidly decreases by more than 7 orders of magnitude. This causes a simple impact algorithm to fail since the maximum precision of the Newton's method code will be exceeded. To compensate for this, the simulation assumes that values of Δt_i below 10^{-7} indicate that the tip mass has stuck on the moving table (i.e., $v_{10} - v_T = 0$). Newton's method is then used to solve for the time when the reaction force between the tip mass and table passes through zero (decreasing), thus indicating that the tip and table will lose contact.

Sticking events are an essential aspect of impact oscillator dynamics, not *ad hoc* assumptions use to make the simulations function properly. Defining the ratio between successive impact time intervals to be

$$\lambda_i = \frac{\Delta t_i}{\Delta t_{i-1}} ,$$

one sees (Fig. 3) that λ_i converges to a value less than 1 during a sticking event (until the precision of the calculation is exceeded, after about 90 impacts). This indicates that the ratio test is satisfied and thus $\Sigma \Delta t_i < \infty$; that is, sticking occurs *in finite time*. Most interesting is the indication that $\lambda_i \rightarrow e$ as $i \rightarrow \infty$, just as occurs for the elementary example of a ball bouncing on a stationary surface. Thus, this study demonstrates that periodic solutions in which sticking occurs possess an infinite number of impacts over a finite number of table periods and are thus of type period-∞, *m*.

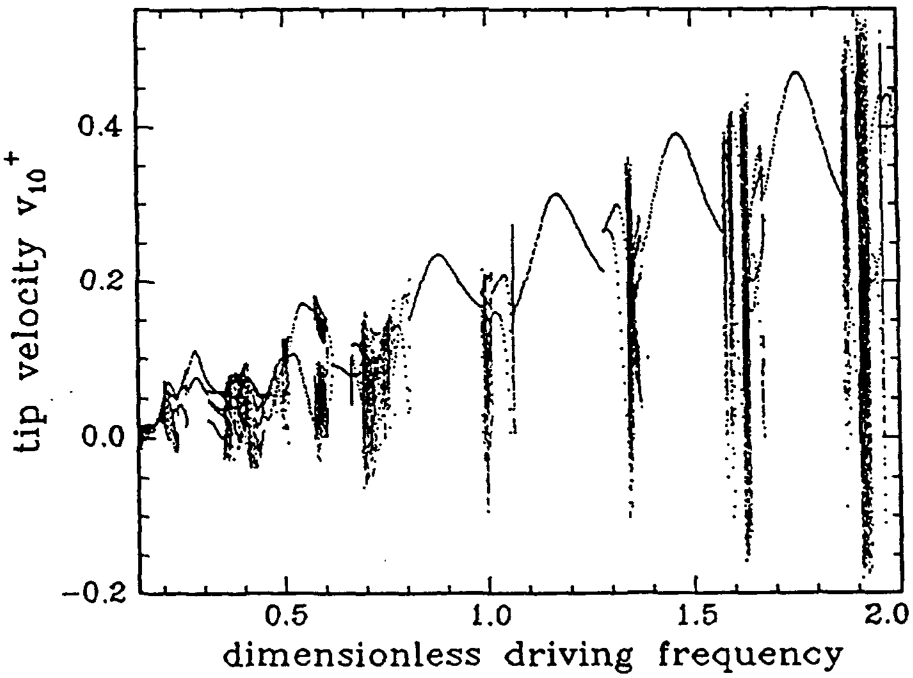

Fig. 1 Impact map bifurcation diagram over whole natural frequency range of the system

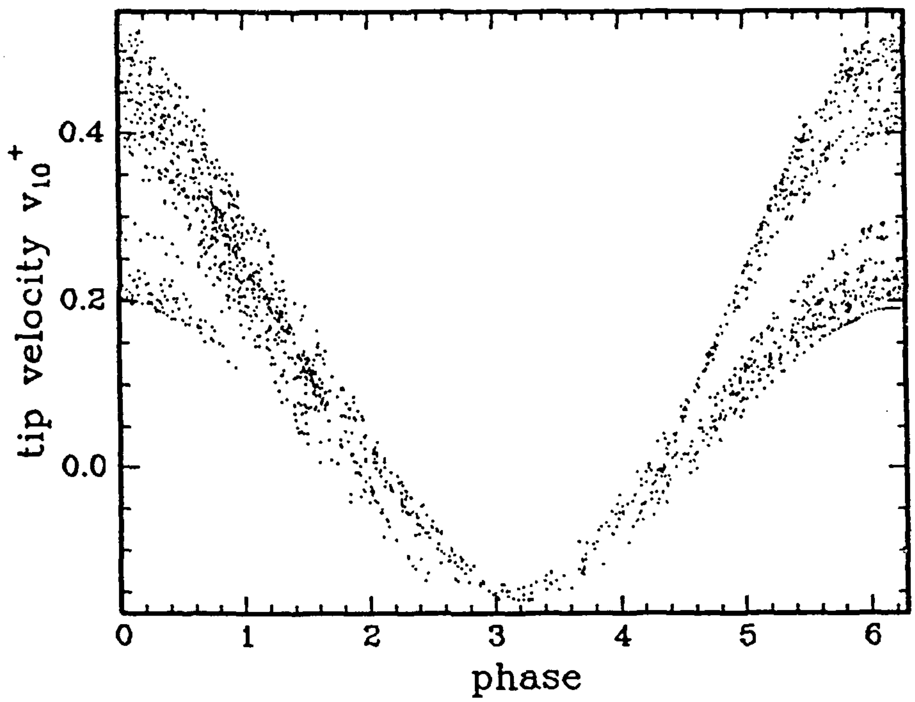

Fig. 2 Poincaré Map of a Chaotic Motion

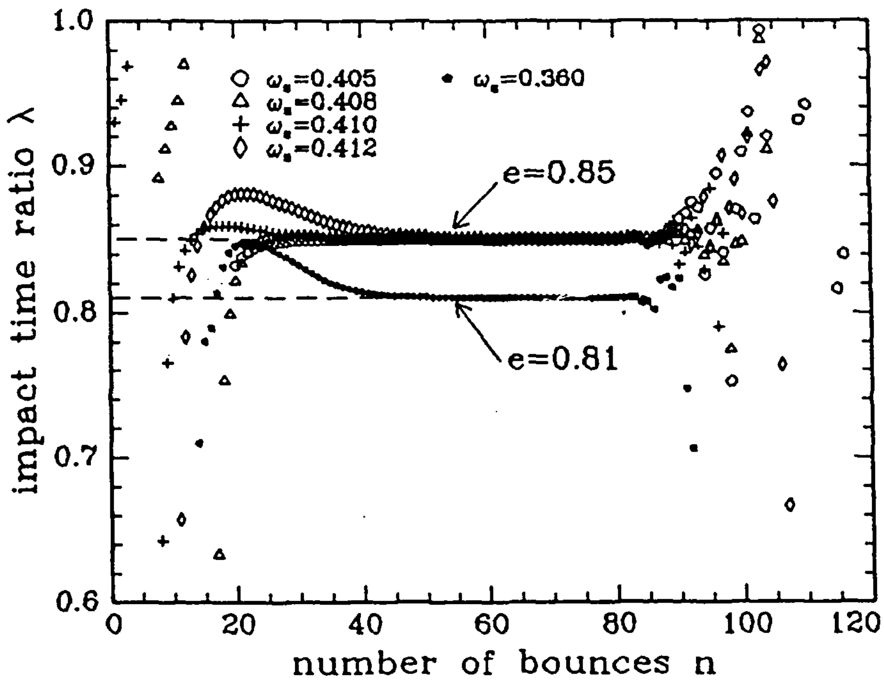

Fig. 3 Convergence of impact time ratio λ for a sticking event.

CONTINUOUS CONTACT FORCE MODELS

FOR IMPACT ANALYSIS IN MULTIBODY SYSTEMS

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ABSTRACT

For the impact analysis between bodies of a multibody system, two types of analyses are normally considered: piecewise or continuous. In a piecewise analysis, the equations of motion are integrated until the moment of contact, then the change in the system momenta and, hence, the discontinuities in the velocities are determined. After updating the velocities, the integration of the equations of motion is resumed. In a continuous analysis, however, the integration of the equations of motion is not interrupted during the period of contact, therefore, a model for evaluating the contact force is required. In this paper, two such continuous contact force models are presented. Both models are of Hertzian nature and are based upon the direct central-impact of two solid particles.

When the local plasticity effects are the dominant factor accounting for the dissipation of energy in impact (at high impact velocities), a Hertzian contact force model with permanent indentation is constructed. Utilizing energy and momentum considerations, the unknown parameters in the model in terms of a given coefficient of restitution and velocities before impact are analytically evaluated. The equations of motion of the two solids are then integrated forward in time as a function of the variation of the contact force during the contact period.

At lower impact velocities, the energy dissipation during impact is mostly due to material damping and not permanent indentation. Based on the general trend of the Hertz contact law, a hysteresis damping function is encorporated into the model which represents the dissipated energy during impact. The unknown parameters in the model are again determined in terms of a given coefficient of restitution, and the validity of the model is established.

The two particle models are generalized to the impact analysis between two bodies of a multibody system. The concept of effective mass is presented in order to compensate for the effects of joint forces. The impact analysis for several examples are presented and the results are compared.

SESSION 4
DYNAMICS AND CONTROL
MONDAY - 1030 - 1210
JUNE 8, 1992

A Singular Perturbation Analysis of the Equations of Servo-Hydraulics

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Servohydraulic drive systems provide excellent dynamic capabilities and have a wide range of applications (e.g.: in material testing machines, for the excitation in experimental simulation of vehicle dynamics, as actuators in rolling mills, air-planes, etc.). Besides the supply unit, which has to provide constant system pressure over the whole range of required flow rates, such a system comprises typically (see fig. 1)

- a servo valve,
- a hydraulic cylinder or a hydro motor, and
- a closed loop control system.

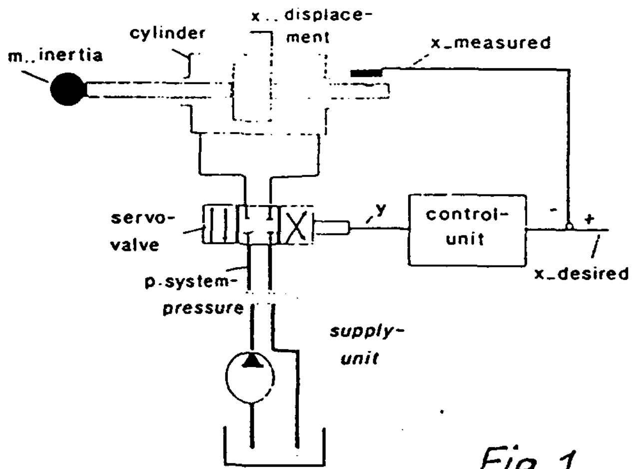

Fig. 1

A simple mathematical model of such a hydraulic drive consists of a system of ODE's. The pressures in both chambers of the cylinder, the displacement of the piston (see fig. 1), and variables of the control unit are used as dependent variables. The equations are strongly nonlinear and singularly perturbed. The nonlinearity arises by the pressure loss in the servo valve, which is quadratic with respect to the flow rate. The singular perturbation is due to the low compressibility of the hydraulic fluid and occurs in the evolution equations for the pressures.

The mathematical investigation found in the engineering litarture is characterized by the application of linear methods of control theory, mainly the Laplace transform method.

In the current analysis, which is restricted to periodic movements of the piston, singular perturbation techniques are applied, to obtain a good understanding of the qualitative features of the system behaviour without prior simplifications like linearization. Numerical investigations can be performed, if solvers for stiff systems are used. But besides the problem of time consuming calculations, which occur particularly, if the frequency of the given movement of the piston is relatively low, they cannot provide compact analytical formulas for design purposes, which are extremely helpful in practice.

A two step approach is made. In the first, the system is analysed without a closed loop control, thus the valve opening is a given periodic function of time. This should provide the understanding of the qualitative behaviour of the controlled system. The main question is, whether internal layers do occur at those instances, when the valve passes through its zero position. The reduced system (compressibility of the fluid set to zero) is singular at this zero valve opening position. A solution is given by a power series expansion and it turns out, that for practical applications, internal layers do not occur. This is bacause of the limited response time of currently available servo valves.

In a second step a closed loop control is added. It is shown, that in contrast to competetive drive systems (e.g. electric drives), where a force or torque and consequently an accelleration is the controller output, for the hydraulic system the speed is the directly regulated quantity. This advantage in the order of the controlled quantity explains, why a rather simple control of P-response type gives satisfactory results in many practical applications.

Throughout the analysis the mathematical manipulation language MAPLE V is used, to perform series expansions, to solve differential equations, and to produce graphical representation of results obtained.

Convergence and Asymptotic Stability of Hamiltonian Control Systems

Part (I): High Gain Tracking Control

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ABSTRACT

Asymptotic stability of high gain control schemes have been dervived for linear systems, as well as nonlinear, first order systems that are asymptotically stable in the first approximation. This paper presents sufficient conditions for the asymptotic stability of output feedback for a class of nonlinear Hamiltonian control systems. In this paper it is shown that rigorous upper bounds can be found for the norm of the error in tracking control in terms of a gain normalization factor. Output feedback control having a magnitude of $O(1/\varepsilon)$ guarantees that the tracking error, and its derivatives, remain in a hyperellipsoid in phase space having a diameter of $O(\varepsilon)$ for the class of systems considered. In addition, the asymptotic convergence, i.e. approximate controllability, of the method is investigated using LaSalle's invariance principle. It is shown that as $t \rightarrow \infty$, the tracking error, and its derivative, approaches the surface of the hyperellipsoid.The theoretical results presented in this paper are validated in a numerical example. Furthermore, the results presented herein form the basis for adaptive Hamiltonian control methods presented in part (II) of the paper.

(1) INTRODUCTION

The study of control design methodologies for nonlinear dynamical systems has become the focus of some of the most significant research in control theory over the past few years. One recent approach for the design of controllers for nonlinear systems employs Lie Algebraic techniques to achieve canonical nonlinear systems for which control design is simpler. Overviews of Lie algebraic approach are given in [4,5,7], while typical specific applications of the method are described in [4]. The advantages to the Lie Algebraic formalism are numerous:

(i) It has a strong foundation in geometric methods that offer valuable insight to the formulation.

(ii) The method is general, and defines controllability and observability concepts for nonlinear systems that are natural extensions of the more common definitions for linear systems.

(iii) The theory enables explicit formula to be derived to achieve exact linearization, asymptotic stabilization and output tracking for some classes of nonlinear systems.

Still, there are attributes of the approach that make it difficult to use for systems with many degrees of freedom. For example, the calculation of relative degree or linearizing observers [7], can be algebraically intractable. Furthermore, for mechanical or structural systems, the equations of motion appear most naturally as Lagrangian, second order systems of ordinary differential equations, or via a hamiltonian formulation. The former of these formulations must be converted to first order form to employ most of the more common Lie Algebraic methods directly, while the second class has additional symplectic structure.

For these reasons, this paper explores nonlinear control using a second popular approach: The Hamiltonian control formulation [10-13]. While the Lie Algebraic methods and Hamiltonian formulations are hardly mutually exclusive (for example one can see [7] or [14]) Hamiltonian formulations can have two distinct advantages

(i) The additional structure of Hamiltonian mechanics enables additional, sharper results in many cases.
(ii) The equations governing mechanical and structural systems are relatively easily obtained in Hamiltonian formulations.

This paper shows that an asymptotically stable output tracking control can be constructed for a class of nonlinear, multi-input/multi-output Hamiltonian control systems. The approach discussed herein generalizes the class of feedback considered in [7,14] for Hamiltonian systems. It is shown that for the class of systems considered a feedback control having a magnitude of $O(\frac{1}{\varepsilon})$ constrainsthe tracking error to a ball of radius $O(\varepsilon)$ in phase space. In this sense, theorem (3.1) of this paper can be construed as a high gain output tracking theorem analagous to that in [4] for linear and nonlinear systems that are asymptotically stable in the first approximation. Finally, the approach taken in this paper characterizes the ω-limit set of the closed loop tracking error for cases in which the internal energy approaches a constant. The ω-limit set is contained in a hyperellipsoid in phase space of diameter $O(\varepsilon(H_o(O) - H_o(\infty)))$ where H_o is the internal energy.

NEW COMPOSITE ADAPTIVE CONTROL ALGORITHM OF RIGID ROBOTS

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The convergence rate and robust properties of the existing adaptive control schemes of rigid robots can be enhanced by improving the identifiability of adaptive algorithms. We present a new adaptive control algorithm of rigid robots which provides the global exponential convergence of the estimated parameters to their true values for persistently exciting desired trajectories . Our control law does not require acceleration measurements nor does it require the filtering of the velocity signal. New form of the Lyapunov function was found . The results are confirmed by simulation of two link planar manipulator .

Based on Lagrangian formulation , the motion equations of rigid n-link manipulator are as follows :

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau , \qquad (1)$$

where $q \in R^n$ is the vector of joint coordinates , $M(q)= M(q)^T > 0$ is the inertia matrix , $C(q,\dot{q})$ is the matrix of Coriolis and centrifugal torques , $G(q)$ is the vector of the gravitational torques , $\tau \in R^n$ is the vector of input torques .

Three specific properties of the equation (1) we shall use for algorithms design . The first property is following : both the inertia matrix and the inverse of the inertia matrix have the upper bounds ($\sigma_1 I_n \leqslant M(q) \leqslant \sigma_2 I_n$, $\sigma_1 , \sigma_2 > 0$ for all $q \in R^n$). Secondly , in the case of proper definition of the matrix $C(q,\dot{q})$ the matrix $\dot{M}(q) - 2\, C(q,\dot{q})$ is skew-symmetric matrix (Koditschek , 1984) . The third property is linear parameterization property , i.e. the manipulator dynamics is linear in terms of suitably selected set of equivalent manipulator parameters (Slotine and Li ,1989).

Our problem is following : to find the control and adaptation laws for the manipulator with unknown parameters such that to achieve the next control aim

$$\lim_{t\to\infty} \tilde{q}(t)=0 \quad , \qquad \lim_{t\to\infty} \dot{\tilde{q}}(t)= 0 \ , \tag{2}$$

where $\tilde{q}(t)=q(t)-q_d(t)$, $q_d(t) \in R^n$ is the desired trajectory . Define the error vector function $s(t) = \dot{\tilde{q}}(t) + \Lambda\, \tilde{q}(t)$, where $\Lambda = \Lambda^T > 0$. Set the secondary control aim

$$\lim_{t\to\infty} s(t)=0 \ . \tag{3}$$

Consider the following control law

$$\tau = \tilde{Y}(q,\dot{q},\dot{q}_r,\ddot{q}_r,s)\ \theta - \alpha\ s - K(q,\dot{q})s \ , \tag{4}$$

where $\tilde{Y}(\dot{q},\dot{q},\ddot{q}_r,\dot{q}_r,s) = Y(\dot{q},\dot{q},\ddot{q}_r,\dot{q}_r) - Y_1(\dot{q},q,s)$, $Y(\dot{q},\dot{q},\ddot{q}_r,\dot{q}_r)\ \theta = \hat{M}(q)\ \ddot{q}_r + \hat{C}(q,\dot{q})\ \dot{q}_r + \hat{G}(q)$, $Y_1(q,\dot{q},s)\ \theta = (\hat{\dot{M}}(q) - \hat{C}(q,\dot{q}) + \alpha_0\hat{M}(q))s$,

where $\hat{M}(q)$, $\hat{C}(q,\dot{q})$, $\hat{\dot{M}}(q)$ $\hat{G}(q)$ are the estimates of $M(q)$, $C(q,\dot{q})$,$\dot{M}(q)$ matrices and vector $G(q)$ respectively , $\dot{q}_r(t)= \dot{q}_d(t)- \Lambda\ \tilde{q}(t)$ is the virtual desired trajectory , $\theta \in R^m$ is the vector of adjustable parameters. $K(q,\dot{q}) \geqslant -1\backslash 2\ \dot{M}(q)$ for all q, $\dot{q} \in R^n$ and $\alpha \geqslant 3/2\ \alpha_0\sigma_2{}^2$. α_0 is a positive number .

$$\dot{\theta} = - \Gamma(t)[\ \tilde{Y}^T(q,\dot{q},\dot{q}_r,\ddot{q}_r,s)\ s(t) - \alpha_0\varphi(t)\varepsilon(t)], \tag{5}$$

where an $m \times n$ matrix $\varphi(t)$, $\varepsilon(t) \in R^n$ and an $m \times m$ gain matrix $\Gamma(t)$ are adjusted as follows :

$$\dot{\varphi}(t) = -\alpha_0\varphi(t) + \tilde{Y}^T(q,\dot{q},\dot{q}_r,\ddot{q}_r,s) \;, \tag{6}$$

$$\dot{\varepsilon}(t) = -\alpha_0\varepsilon(t) - \alpha\, s - K(q,\dot{q})\, s + \tag{7}$$

$$\varphi(t)^T\Gamma(t)\tilde{Y}^T(q,\dot{q},\dot{q}_r,\ddot{q}_r,s)s - \alpha_0\varphi(t)^T\Gamma(t)\varphi(t)\ \varepsilon(t) \;,$$

$$\dot{\Gamma}^{-1} = \alpha_0\, \varphi\, \varphi^T/2 - \alpha_0\lambda(t)\Gamma^{-1}/2 \;, \Gamma(0)=\Gamma(0)^T > 0 \;, \|\Gamma(0)\| \leqslant k_0 \;,$$

$$\lambda(t) = \lambda_0\,(1 - \|\Gamma\|/k_0), \tag{8}$$

where , $\lambda(t)$ is a variable nonnegative forgetting factor, λ_0 and k_0 are two positive constants. It should be noted that least-squares gain update (8) with time-varying forgetting factor was proposed by Slotine and Li (1989) and called "bounded-gain forgetting method".

With the preceding control structure we can prove the following

Theorem 1.Consider the system (1),(4) with adjustment law (5)-(8) .If all desired trajectories are bounded then

i) the secondary control aim (3) is achieved ,

the main control aim (2) is achieved

and all trajectories of the system remain bounded .

ii) If, in addition , $\varphi(t)$ is persistently exciting than the tracking errors and the estimation errors exponentially converge to zero .

The proof is based on the Lyapunov function

$$V = 1/2\ s^T M(q)s + 1/2\ \| M(q)s - \varepsilon - \varphi^T\tilde{\theta} \|^2 + 1/2\ \| \tilde{\theta} \|^2_{\Gamma(t)^{-1}} \;.$$

where $\tilde{\theta} = \theta - \theta_*$, θ_* is the vector of true parameters .

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Slewing Dynamics and Control of the Space Station Based Mobile Servicing System

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A relatively general formulation for studying the nonlinear dynamics and control of spacecraft with interconnected flexible members in a tree-type topology is developed. The distinctive features of the formulation include the following: (i) it is applicable to a large class of present and future spacecraft with flexible beam and plate type appendages, arbitrary in number and orientation; (ii) the members are free to undergo predefined slewing maneuvers to facilitate modelling of sun tracking solar panels or large angle maneuvers of space based robots. Besides, the environmental disturbances due to thermal deformations of flexible members are incorporated in the study; (iii) the governing equations of motion are highly nonlinear, nonautonomous and coupled. They are programmed in a modular fashion to help isolate the effects of flexibility, librational motion, thermal deformation, slewing maneuver, shifting center of mass, higher modes, initial condition, etc.

Next, versatility of the general formulation is illustrated through the parametric analysis of the Mobile Servicing System (MSS) to be developed by Canada for operation on the Space Station. The MSS is essentially a two link manipulator attached to a mobile base which traverses along the station power boom. The functions of the MSS would not be limited to satellite retrieval and release. Instead it is expected to be the workhorse for the station construction, maintenance, and operations. The MSS, being flexible in the links as well as the joints, is an extremely complicated system to study. The objective here is to assess pointing errors arising from inplane and out-of-plane maneuvers of the robotic arms. Finally, the attention is focused on development of control strategies to restore equilibrium. To that end, feasibility of the nonlinear control based on the Feedback Linearization Technique (FLT) is explored. Results show the procedure to be quite promising in controlling the attitude of the space station over a range of disturbances arising from MSS maneuvers.

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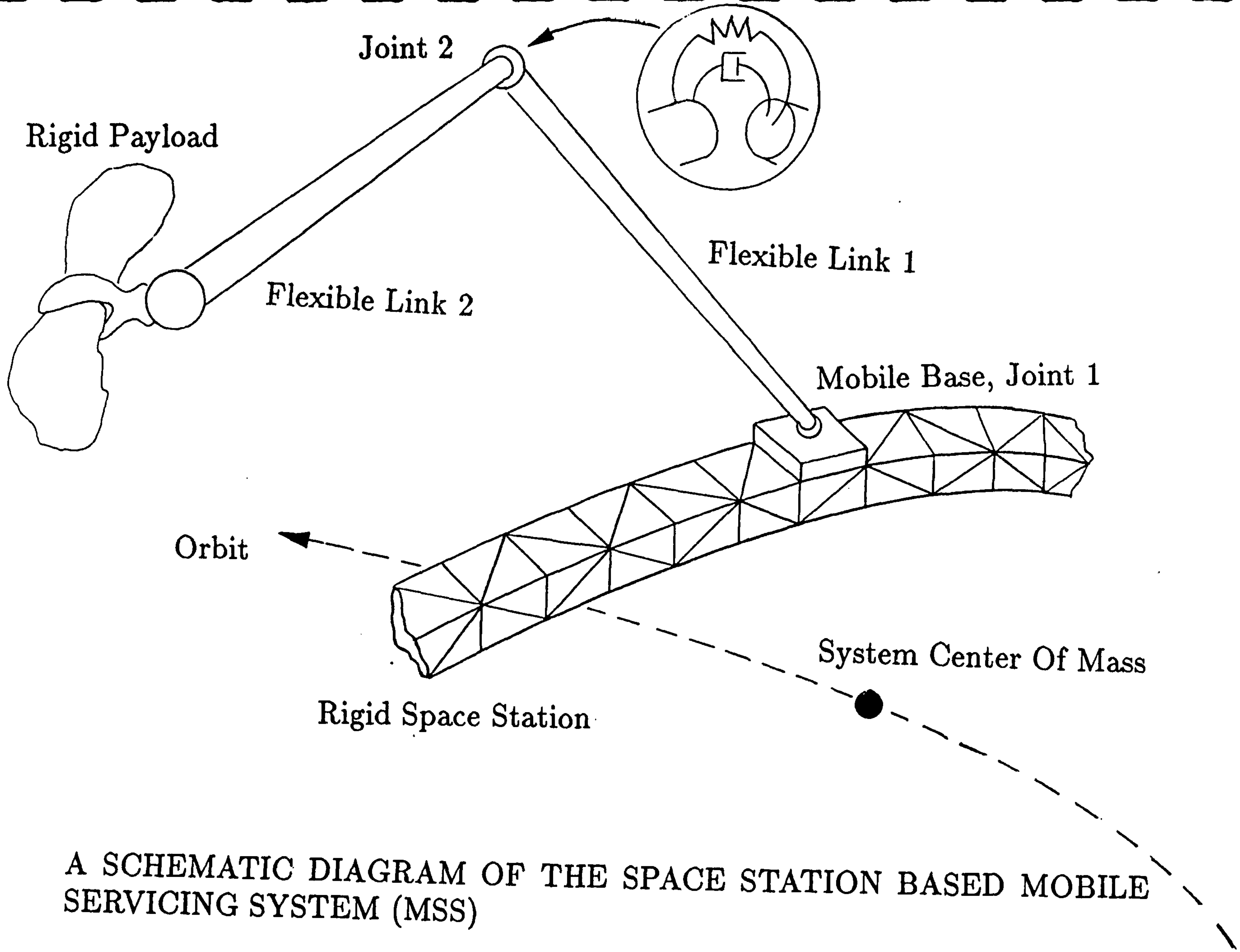

A SCHEMATIC DIAGRAM OF THE SPACE STATION BASED MOBILE SERVICING SYSTEM (MSS)

EFFECT OF THE LINEARIZATION OF THE CORIOLIS AND CENTRIFUGAL FORCES ON THE FEEDFORWARD CONTROL LAW OF FLEXIBLE MECHANICAL SYSTEMS

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ABSTRACT

The inverse dynamics problem for articulated structural systems such as robotic manipulators is the problem of the determination of the joint actuator forces and motor torques such that the system components follow specified trajectories. In many investigations, the open loop control law was established using an inverse dynamics procedure in which the centrifugal and Coriolis inertia forces are linearized such that these forces in the flexible manipulator model are the same as those in the rigid body model. In some other investigations, the effect of the nonlinear centrifugal and Coriolis forces is neglected in the analysis and control system design of articulated structural systems. It is the objective of this investigation to study the effect of linearization of the centrifugal and Coriolis forces on the nonlinear dynamics of constrained flexible mechanical systems. The virtual work of the inertia forces is used to define the complete nonlinear centrifugal and Coriolis force model. This nonlinear model that depends on the rate of the finite rotation and the elastic deformation of the deformable bodies is used to obtain the solution of the inverse dynamics problems, thus defining the joint torques that produce the desired motion trajectories. The effect of the linearization of the centrifugal and Coriolis forces on the obtained feedforward control law is examined numerically using different sampling rates and different number of vibration modes. The results presented in this investigation is obtained using a slider crank mechanism with a flexible connecting rod.

SESSION 5

APPLICATIONS TO CABLES, STRINGS, AND ROTORS

MONDAY - 1330 - 1510

JUNE 8, 1992

TWO-TO-ONE INTERNAL RESONANCE IN SUSPENDED ELASTIC CABLES

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This study examines the near-resonant response of suspended, elastic cables driven by planar excitation in the presence of a two-to-one internal resonance. A geometrically nonlinear continuum cable model is presented which describes three-dimensional response. An asymptotic form of the model, representing suspensions with small equilibrium curvature (sag) and horizontal supports, is discretized using the Galerkin method. The resulting two-degree-of-freedom discrete model is used to examine the coupling between a symmetric in-plane mode and an out-of-plane mode. These modes are coupled through quadratic and cubic nonlinearities which originate from nonlinear cable stretching. Two-to-one internal resonances naturally arise for specific sag levels where the natural frequency of the in-plane mode is, approximately, twice that of the out-of-plane mode. Planar and non-planar response of the cable is examined for conditions near primary resonance of the in-plane mode.

A perturbation analysis using a version of the method of multiple scales is carried out to second nonlinear order to examine the existence and stability of weakly nonlinear periodic motions. At the first nonlinear order, the discrete model is shown to possess the particular quadratic nonlinear terms that lead to saturation. As shown in Figure 1(a), the directly excited mode (a_1) saturates beyond the excitation amplitude, $F = 5.22 \times 10^{-4}$. In the excitation range, $5.22 \times 10^{-4} < F < 7.17 \times 10^{-4}$, two stable periodic responses co-exist: one a planar (a_1 only) response and the other a coupled response.

The analysis is then extended to second nonlinear order to capture the additional effects of the cubic nonlinearities and to include higher order corrections due to the quadratic nonlinearities. Figure 1(b) illustrates that the saturation phenomena is disrupted and that higher order effects may qualitatively alter the nature of the steady state response. Here, a_1 never saturates but monotonically increases with increasing F. Furthermore, the higher order corrections split the (formerly degenerate) stable and unstable a_1 solution branches in the muti-response region.

The accuracy of the higher order solutions are verified by comparison to results obtained by numerically integrating the equations of motion (diamonds).

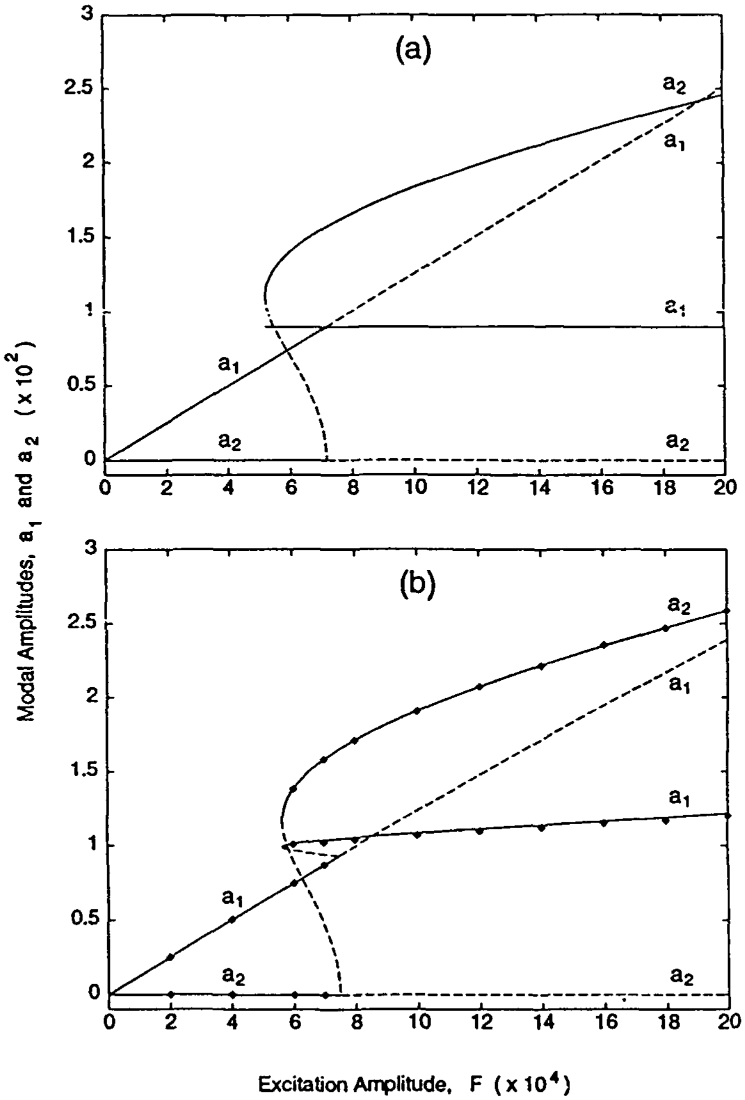

Figure 1

TRANSIENT VIBRATIONS AND CONTROL OF A TAUT INCLINED CABLE WITH A RIDING ACCELERATING MASS

by

Iradj Tadjbakhsh[1] and Yi–Ming Wang[2]

ABSTRACT

The dynamics and the numerical solution for vibrations of a taut inclined cable and the motion of a riding accelerating mass is developed. The moving mass is a trolley that is accelerated by a solid fuel racket down the inclined cable to very high velocities and is aerodynamically brought to a hault. The thrust of the rocket is tangential to the deformed shape of the cable.

The mechanics of the problem is Newtonian. Method of analysis consists of small deformations superimposed on the static catenary state. The problem is nonlinear due to convective acceleration interactions of the moving mass and the cable as well as the presence of friction.

To reduce amplitudes of vibrations in the cable the method of vibrational control is employed. According to this method the tension in the cable is varied by sinusoidal increment controlled at one end. The amplitude and the frequency of the incremental tension is adjusted in order to reduce the level of response.

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TRANSIENT VIBRATIONS AND CONTROL OF A TAUT INCLINED CABLE WITH A RIDING ACCELERATING MASS

Numerical solution uses Galerkin's procedure to remove spatial dependence and to reduce the problem to a nonlinear finite–dimensional state–space representation which is solved as as an initial value problem. The results include the shape of the cable at any instant and the kink angle which is the angle between the tangents to the cable behind and ahead of the trolley.

Modal Interactions in a Parametrically and Externally Excited String

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In this paper, we present theoretical and experimental results for a stretched string subjected to a planar simple-harmonic excitation at one of its ends. A schematic diagram of the experimental setup is shown in Figure 1. The shaker is oriented to provide an excitation with components both in the direction of and transverse to the axis of the string in order to generate a combination of parametric and external excitations.

It has long been known that under certain conditions a string subjected to a transverse planar force will whirl like a jump rope [1]. The reported whirling motions are a result of the nonlinear coupling between the in-plane and out-of-plane modes of the string due to the variation in the tension in the string as it deflects. Several recent studies have focused on quasiperiodic and chaotically-modulated motions which may occur as the string whirls [2]-[5]. None of these studies, however, include the effects of a parametric excitation.

The transverse motion of the string is governed by two nonlinear coupled partial-differential equations [6]. We apply the method of multiple scales directly to these governing equations and their boundary conditions to obtain a set of nonlinear coupled ordinary-differential equations governing the amplitudes and phases of each of the in-plane and out-of-plane modes of the string. We show that when the parametric excitation is set to zero only the in-plane and out-of-plane modes at the frequency of the excitation have nontrivial long-time behavior.

When the external excitation is set to zero, only modes at one-half the excitation frequency can be excited. If the excitation frequency is near the natural frequency of an even-numbered mode, the parametric excitation can excite the mode at half that frequency whereas if the excitation frequency is near that of an odd-numbered mode, no modes will be excited. When both the parametric and external excitations are present and the excitation frequency is near the natural frequency of an even-numbered mode, the modes at half the frequency of excitation as well those at the frequency of excitation can be excited.

With the parametric excitation set to zero, our analysis predicts that periodic whirling motions can occur. This is confirmed experimentally and a typical periodic whirling motion is shown in Figure 2. A linear analysis of the stability of this solution shows that as the frequency of excitation is varied, the periodic whirling motion may become unstable, giving rise to complicated modulated motions. These results are in agreement with those in references [1] through [5].

A combination of parametric and external excitations can lead to whirling motions where modes at the excitation frequency as well as modes at half the excitation frequency can be excited. In this case, periodic whirling motions appear with a 'figure 8' shape as shown in Figure 3. Under certain conditions, these motions may also lose stability giving rise to complicated modulated motions. The experimental results are in good agreement with the theory.

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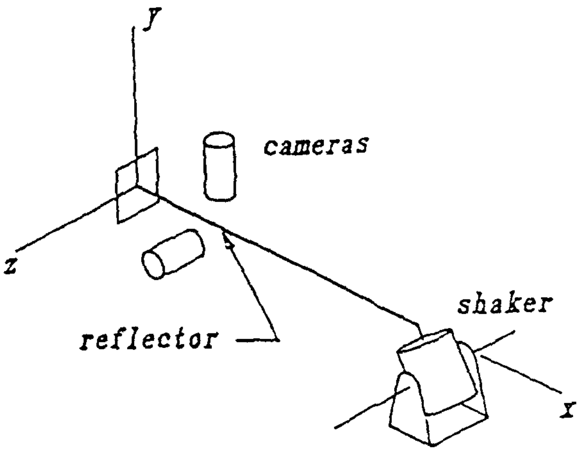

Figure 1 Experimental setup

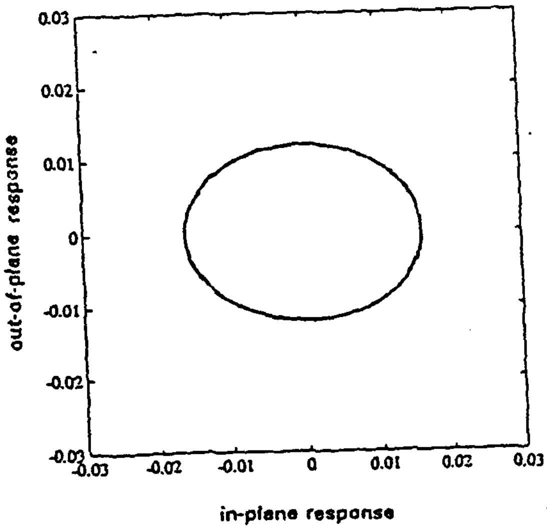

Figure 2 Periodic whirling response to external excitation

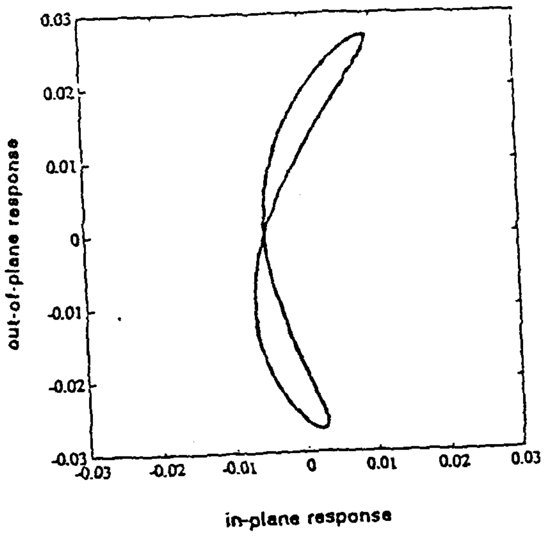

Figure 3 Periodic whirling response to parametric and external excitation

BEHAVIOR OF A CRACKED ROTATING SHAFT DURING PASSAGE THROUGH A CRITICAL SPEED

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There has been extensive research on the vibrational behavior of cracked shafts and the use of response characteristics to detect cracks (e.g., see the reference lists in Wauer [1] and Collins *et al.* [2]). Most of this previous work involved shafts rotating at constant angular speed, often focusing on the changes of the natural frequencies or modes. However, Wang *et al.* [3] and Bently and Thomson [4] noted that it is easier to detect cracks during a start-up or run-down process than at a steady speed. According to Nilsson [5], cracks are usually more evident when a rotor passes through a resonance than under normal operation. Vibration monitoring during run-down is sometimes used in an attempt to detect cracks, but research on transient responses of cracked shafts has been limited.

In the present study, transient responses of a horizontal, simply supported, rotating shaft are studied analytically. Euler-Bernoulli theory is applied. Torsional and longitudinal vibrations are neglected, whereas forces due to eccentricity, gravity, and internal and external damping are included. The shaft contains a single transverse crack that is assumed to be either completely open or completely closed at any given time, depending on the curvature of the shaft at the cross section containing the crack. Results for this breathing crack are compared to those for a crack that is always open and to those for an uncracked shaft.

The governing equations of motion are bilinear. Galerkin's method is utilized with five-term approximations for the two displacement functions, and the resulting equations are integrated numerically. Natural frequencies and critical speeds are determined for the unforced, undamped shaft with an open crack and with no crack. Then time histories of the response are computed when the shaft is accelerated or decelerated past the fundamental critical speed at a constant rate. The maximum response is determined, and the effects of the acceleration and deceleration rate, crack depth, crack position along the shaft, and eccentricity angle (with respect to the crack face) are investigated.

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Chaotic Motions and Fault Detection in a Cracked Rotor

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A detailed study of the vibrational behaviour of cracked rotating shafts is an important problem for engineers working in dynamics of machines. The cracked mechanical system is described by

$$\mathbf{M}\,\ddot{\mathbf{z}} + \mathbf{D}\,\dot{\mathbf{z}} + \mathbf{K}\,\mathbf{z} = \mathbf{g}(t) + \mathbf{N}_n\mathbf{h}(\mathbf{z}(t)) \tag{1}$$

where $\mathbf{z}$ is the displacement vector, $\mathbf{M}, \mathbf{D}, \mathbf{K}$ are mass, damping and stiffness matrices, $\mathbf{g}(t)$ is a vector of unbalances, $\mathbf{N}_n$ is an input matrix of nonlinearities and $\mathbf{h}$ is a vector of disturbances caused by the crack, which describes the change in stiffness coefficients. So the system becomes one that is parameter excited and nonlinear (with discontinuieties caused by transitions between motion without crack and motion with crack).

In this paper two kinds of problems are considered:

1) Vibrational behaviour of the cracked turbo rotor depending on the parameters, i.e. the crack coefficient and the damping coefficient.

2) A new observer based method for detection of cracks in turbo rotors.

A) Depending on the system parameters, i.e. crack depth or damping coefficients, different types of motion are obtained: periodic, almost-periodic, sub- or ultraharmonic, and even chaotic ones. Several quantitative and qualitative measures for the characterization of attractors exist, e.g. such of phase plane plots, FFT-analysis, different kinds of dimensions and entropies. Lyapunov exponents are chosen here to classify the system behaviour. In the case of single time series only the greatest Lyapunov exponent is calculated. To obtain the spectrum of Lyapunov exponents from differential equations (1) the linearized equations are needed. It is shown that the general method can be applied if some care is devoted to the handling of the discontinuities. These equations have to be supplemented with transition function conditions, and then allow one to determine the spectrum of Lyapunov exponents in the case of nonlinear systems with discontinuities.

B) The clear relation between shaft cracks in turbo rotors and vibrational phenomena measured in bearings can be established by model- based methods very well. Here the new method [1,2] based on the theory of disturbance rejection control, extended for non-linear systems and applied to a turbo rotor is presented as well. In this way the crack is interpreted as an external disturbance. Due to the theory of estimating unknown disturbances of a dynamic system, simple measurements of displacements and/or velocities - obtained from simulation or experiment - are used to reconstruct these additional time signals by state observers to obtain estimates of the non- linear effects. The state observer is based on the known part of the vibration system and a linear fictitious model, which approximates the crack. By calculating the relative crack compliance as the ratio of additional compliance caused by the crack and undamaged compliance, a clear statement is possible about the opening and closing, and therefore for the existence of the crack, and about the crack depth.

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SESSION 6
OPTIMIZATION AND COMPUTATIONAL METHODS
MONDAY - 1530 - 1710
JUNE 8, 1992

An efficient algorithm for elasto-viscoplastic vibrations of multi-layered composite beams using second-order theory.

By

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Abstract:

Engineering structures composed of layers are characterized by both light weight and high strength. This is due to modern technologies, allowing a problem-oriented choice of the material properties in the different layers. The present paper is concerned with a time-domain algorithm for plane non-linear flexural vibrations of composite beams, which are driven into the inelastic range by severe transverse loadings. For a straight-forward structural analysis, the non-classical coupling between flexure and stretching due to an unsymmetric combination of the layers in the thickness-direction is resolved by a proper choice of the position of the beam axis. The influence of an axial static preload is considered in the sense of the quasi-linear second-order theory of structures, namely by applying the equations of momentum to the deformed beam but using linearized geometric relations. The main goal of the paper is the appropriate incorporation of viscoplastic strains into such a second-order theory of composite beam vibrations. This is achieved by means of an efficient semi-analytic algorithm: The inelastic parts of strain are treated as additional sources of selfstress in the linear elastic background-structure, driving the elastic response into the inelastic one. Those sources act likewise to a fictitious time-dependent thermal loading in the associated linear elastic composite beam. The efficiency of this exact formulation lies in the fact that well-known linear

solution techniques can be used in their most powerful form, because the stiffnesses of the associated linear beam are time-invariant. In the present paper, the second-order transfer-matrix technique in combination with modal analysis is adopted for multi-span beams. Accelerated convergence of modal expansions is obtained by means of a splitting of the total solution into a quasi-static and a dynamic part. The fictitious thermal loading, which occurs in this analytic formulation as an additional driving term, is calculated in a time-stepping procedure as the inelastic parts of strain using the local non-linear material laws of the different layers. Any non-linear numerical solution routine may be used, which is appropriate for the specific mathematical type of the material laws. Having calculated the inelastic strain increments, the increments of the overall beam variables are obtained using the above dynamic version of the transfer-matrix technique. For an efficient application of this scheme, the effects of the fictitious thermal loading upon the dynamic behaviour are evaluated in the sense of the dynamic influence function method at once before starting the time-stepping procedure. Furthermore, the time-stepping is performed by using an initial-value storage technique for the increments of the modal coordinates in order to hold the numerical effort equally low during the course of calculations. The procedure is demonstrated for layered beams with overhang and axial compression under the action of a blast-type transverse loading. Layers with different material behaviour according to the versatile elasto-viscoplastic formulation of Perzyna are considered. The generalized mid-point rule is used as implicit algorithm for the calculation of the viscoplastic strains. Results of the semi-analytic procedure are presented in non-dimensional form as a parameter study.

SINGULARITY-FREE AUGMENTED LAGRANGIAN ALGORITHMS FOR CONSTRAINED MULTIBODY DYNAMICS

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ABSTRACT

After a general review of the methods currently available for the dynamics of constrained multibodies in the context of numerical efficiency and ability to solve the differential equations of motion in singular positions, we examine the acceleration based augmented Lagrangian formulations, and propose a new one for holonomic and non-holonomic systems that is based on the canonical equations of Hamilton. This new one proofs to be more stable and accurate than the acceleration based counterpart under repetitive singular positions. The proposed algorithms are numerically efficient, can use standard conditionally stable numerical integrators and do not fail in singular positions, as the classical formulations do. The reason for the numerical efficiency and better behavior under singularities relies on the fact that the leading matrix of the resultant system of ODEs is sparse, symmetric, positive definite, and its rank is independent of that of the jacobian of the constraints equations. The latter fact makes the proposed method particularly suitable for singular configurations.

In the introduction we review all the current approaches for multibody dynamics that have been proposed in the various fields such as aerospace, robotics and mechanisms. We categorize the methods into those that use recursive procedures, those that use a minimum set of independent coordinates, global methods based on the Lagrange's multipliers approach and those arising from penalty formulations. As background, section 2 describes the classical Lagrange's multipliers formulation and shows how the Hamiltonian formulation leads to better behaved differential algebraic equations than those obtained from the acceleration based

formulation. The same section also describes why the classical formulations fail in singular positions. In section 3, we describe the acceleration based augmented Lagrangian formulation and propose the algorithm ALF1 for the integration of the equations of motion of multibodies in global coordinates.

In section 4 we propose a new augmented Lagrangian formulation that is based on the canonical equations of Hamilton. To this end we define a modified Hamiltonian formulation which is derived by adding to the Lagranges's multipliers terms three penalty terms: a fictitious potential, a set of Rayleigh's dissipating forces and a fictitious kinetic energy term. The resulting equations are subsequently modified to include an integral term that eliminates the numerical stiffness of the original system. A new algorithm ALF2 is then proposed. The numerical results of section 6 clearly show that ALF2 leads to a more stable integration than ALF1. We extend this method to non-holonomic constraint conditions in section 5.

The paper ends with the following conclusions:

- The method is very simple to implement and can use standard off the shelf conditionally stable numerical integrators such as those available in commercial mathematical libraries.
- The fact that the leading matrix of the equations of motion is always positive definite, symmetric and sparse, allows for a very efficient solution of the equations without the use of pivoting. This applies even in the presence of redundant constraints and coordinates, and most importantly in singular positions.
- Although the method is non-recursive, the leading matrix is strongly banded and therefore the number of operations needed to solve the systems of equations becomes of order n.
- The Lagrange multipliers (reaction forces at the constraints) are obtained without having to integrate additional equations.
- The acceleration based formulation ALF1 shows numerical inestabilities under repetitive singular positions that are due to the accumulation of constraint errors. These can be circumvented with tighter tolerances and incresead values in the frequency of the dynamical penalty system at the expense of additional computational cost.
- The canonically based method ALF2 is more robust and has not shown pathological behavior in any of our simulations. These authors do not know of any other algorithm that can simulate the motion of a multibody undergoing repetitive singular positions as ALF2 does.

CONSTRAINED OPTIMIZATION OF SPACE FRAME STRUCTURES

by Jaroslaw A. Czyz* and Stanislaw A. Lukasiewicz**

ABSTRACT

The paper presents optimization methodology for the design of maximum multiple natural frequency space frames subjected to constant volume constrain.

The cross-section of the frame members is assumed rectangular, however, any cross-section defined by two parameters can be considered. Limits for minimum and maximum size are assumed and the constraints for maximum ratio of two dimensions of each cross-section are imposed. The optimization methodology was implemented in automated structural optimization computer code which was used to solve several space frame problems. Numerical results obtained for the selected example problems indicated that the formulation of optimality conditions utilizing separation of bending energy into two orthogonal planes accelerates the convergence of the optimization process.

Structural optimization of three-dimensional space frame type structures called considerable attention of the researchers in recent years. A major effort has been concentrated on the space frame optimization subjected to static and unimodal dynamic constraints.

The present paper is devoted to the optimization of the frame structures made of beams of rectangular cross-sections. The dimensions of the cross-sections of the beams are a_y^i, a_z^i where i = 1, . . . n, and n - is the number of optimized elements. These dimensions are the optimization variables. The beam elements of the structure have a fixed orientation in space and do not change that orientation during the optimization. The optimized structure can consist of other elements, such as, elastic supports, concentrated masses, and elements of constant parameters. The optimization is carried out to find the distribution of geometrical parameters $\bar{a}_y = (a_y^1, a_y^2, \ldots a_y^n)$ and $\bar{a}_z = (a_z^1, a_z^2 \ldots a_z^n)$, for which the fundamental natural frequency reaches the maximum value, while the volume of the structure remains constant. This problem is equivalent to minimization of weight of a frame subjected to multimodal frequency constraints. To find a solution satisfactory from the technical point of view some additional conditions are imposed. It is assumed that the dimensions of the frame cross-sections a_ξ^i ($\xi = y$ or $\xi = z$) are between the limits $a_{min} < a_\xi^i < a_{max}$. Moreover, for each cross-section the ratios a_y^i/a_z^i and a_z^i/a_y^i of one dimension to the other one should not exceed a given value $r_{max} \geq 1$. These constraints are also necessary because of the limitations of the validity of the beam theory.

The maximized fundamental frequency can be multiple, ie., $\omega_1 = \omega_2 = \ldots \omega_m$, where ω_i - is the "*i*th" natural frequency of the frame, m-the multiplicity of the fundamental frequency. It is expected that if the ratio r_{max} is not close to 1, m is at least equal 2 for the optimal structure because of the fact that the ratio of the moments of the inertia of the cross-section of the beam with respect to the *y* and *z* axis is variable.

The solution of the stated problem is achieved by application of the optimality criterion method. An energetic approach to formulation of the optimality conditions allows to separate bending energy in two orthogonal planes of each element (along axis y and z.) The implementation of that approach to the optimization algorithm improves its convergence. The modality of the problem, i.e. the multiplicity of the fundamental frequency of the optimal structure is determined iteratively. Several examples of the solutions are presented.

Optimal Placement via Simulated Annealing of Passively Damped Struts in an Experimental 2-Dimensional Truss

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Before any large structure can be deployed into space, ground modal testing must be performed to determine the dynamic characteristics (i.e., frequencies, modes, and damping) of the structure. These tests might conclude that certain unstable modes of vibration are present and are undesirable for the structure's orbiting maneuvers. In this case, active or passive control must be considered. When deciding where to place passive dampers, their locations are usually assigned on the basis of skilled engineering insight. This method of placement would hold when considering only a single mode of vibration in a simple structure; however, when considering a cluster of modes in a complex structure it is not so apparent where the optimal placement for the passive members should be located. Using a combinatorial optimization approach has proven to have a high computation cost. It then becomes necessary to use a heuristic-based technique that renders near-optimal solutions at a low computation cost.

A study was performed on a two-dimensional twenty bay truss, where motion was restricted to the horizontal plane. When a structure is spacially discrete and a finite number of passive members are to be placed in the structure, an optimal solution can be obtained by using a combinatorial optimization scheme. For the case presented in this study, over

485,000 combinations would have to be calculated to obtain the optimal solution. To avoid such a large amount of computations, a heuristic-based method was examined.

Simulated annealing, a modified iterative improvement algorithm, was used to find a near-optimal solution in a more feasible manner. The simulated annealing algorithm developed by Kirkpatrick et al was based on the analogy between the simulation of the annealing of solids and the problem of solving large combinatorial problems. Simulated annealing allows non-improving solutions to be accepted based on a probability function. This characteristic allows the algorithm to move away from any local optima, unlike the iterative improvement method which has a tendency to get trapped at a local optima.

The control parameters that govern the simulated annealing algorithm were examined and the results are presented. In particular, the control parameter that allows the acceptance of non-improving solutions was modified to enhance the performance of the algorithm. Finally, the search space of the algorithm was restricted to study the effect on the solutions obtained by simulated annealing.

Computation of Lyapunov-Floquet Transformation Matrices for General Periodic Systems

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EXTENDED ABSTRACT

Periodic systems are found in many practical applications such as stability of structures subjected to periodic loading, stability of helicopter rotor blades in forward flight, and control of periodic systems. The equations of motion in general have periodic coefficients and are nonlinear. In many instances, the stability and control problems may be studied via linear equations perturbed about a periodic orbit. Bifurcation studies of the periodic orbit, however, can only be done through nonlinear equations of motion. Therefore, in general all such problems lead to quasi-linear differential equations with periodic coefficients.

$$\dot{x}(t)=A(t)\ x(t)\ +\ f(x,t) \tag{1}$$

where A(t) is T-periodic

It is well known that linear systems with periodic coefficients can be transformed to a time invariant form via the Lyapunov-Floquet transformation. Therefore the above quasi-linear equations can be transformed to

$$\dot{y}(t)=C\ y(t)\ +\ L^{-1}\ f(y,t) \tag{2}$$

where $y(t)\ =\ L(t)\ x(t)$ and C is a time invariant matrix.

Thus, if the $L(t)$ Lyapunov-Floquet transformation matrix (LFTM) could be determined in a symbolic form, the bifurcation and control studies for equation (1) can be reduced to the problem given by equation (2). This problem can be analysed via several techniques available in the literature [1,2].

In order to find the LFTM, one must compute the state transistion matrix associated with the linear part of equation (1) in a form suitable for algebraic manipulations. This is only possible for a very special type of system called a commutative system. For more general systems, it may be possible to employ the perturbation and related methods. These methods, however, are inappropriate for systems where the strained parameter is not small. Numerical techniques are of course unsuitable for this purpose.

Very recently Sinha and Wu [3], and Sinha and Juneja [4] have explored the use of Chebyshev polynomials in the solutions of linear periodic systems. The latter authors have succeeded in computing the state transition matrix in symbolic form. In this study the approach used by Sinha and Juneja [4] is extended to computing the Lyapunov-Floquet transformation matrix and the results are compared with the exact results obtained for a commutative system.

PROPOSED SCHEME:

Consider

$$\dot{x}(t)=A(t)x(t) \tag{3}$$

where A(t) is a periodic matrix with period T. Following references [3] and [4], $A(t)$ and $x(t)$ are expanded in terms of the *shifted* Chebyshev polynomials $T^{*}(t)$. Substituting these expansions in equation (3) and utilizing the proper set of initial conditions, an algebraic form of the state transition matrix $\Phi(t)$ is obtained where each element of $\Phi(t)$ is represented in terms of a set of Chebyshev polynomials. Then $\Phi(t)$ is decomposed into a product of two matrices as

$$\Phi(t)=L(t)e^{tC} \tag{4}$$

where L(t) is T-periodic and C is a constant matrix. L(t) and C are in general complex. Floquet theory guarantees the existance of this decomposition, and further shows that using the following transformation

$$x(t)=L(t)y(t) \tag{5}$$

the original system of differential equations can be transformed to a system with constant coefficients.

$$\dot{y}(t) = C\, y(t) \tag{6}$$

Sinha and Wu have already shown the computational efficiency of using Chebyshev polynomials to find the state transition matrix for a given system with periodic coefficients. The total scheme can be applied to a general system with periodic coefficients to yield a system with a constant linear part and a time varying nonlinear part. This facilitates the use of averaging methods and normal form methods to arrive at a solution. The scheme can also be applied to control problems with periodic coefficients to greatly simplify the system to yield a time invariant system. Several applications of this scheme are demonstrated.

ACKNOWLEDGEMENTS:

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SESSION 7

COUPLED OSCILLATORS I

TUESDAY - 0830 - 1010

JUNE 9, 1992

Whirling of a Forced Cantilevered Beam with Static Deflection: Passage through Resonance

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and

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Virginia Polytechnic Institute and State University

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Non-stationary excitations of slender, elastic, cantilevered beams with equal principal moments of inertia are considered. The excitation frequency is slowly increased or decreased through a resonance of the first mode at a constant rate. Three resonances are investigated: primary resonance, superharmonic resonance of order two and subharmonic resonance of order two. After application of Galerkin's method with three modes, the nonlinear, nonstationary response of the first mode of the beam is determined by two methods: integration of the modulation equations obtained from the method of multiple scales, and direct numerical integration of the temporal equations of motion. Time histories are presented and the effects of excitation amplitude, rate of acceleration or deceleration through resonance, damping and initial conditions of the disturbance on the maximum response are studied. The effect of a persistent random disturbance is also examined. Although the excitation acts in the vertical plane, whirling occurs if the beam is subjected to out-of-plane disturbances.

Forced Oscillations of a Rotating Shaft with Nonlinear Spring Characteristics and Internal Damping
(Subharmonic Oscillation of Order 1/2 and Entrainment)

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School of Engng.,Nagoya University,Furo-cho,Chikusa-ku,Nagoya,JAPAN.

1. Introduction

When an elastic rotating shaft is supported by ball bearings, various kinds of nonlinear forced oscillation appear due to clearance in bearings and a self-excited oscillation appears in the post-critical region due to internal damping which is caused by frictions between the shaft and the inner ring. An entrainment phenomenon in the neighborhood of a resonance point of a sub-harmonic oscillation of order 1/2 of forward whirling mode is discussed.

2. Equations of Motion

An inclination motion of a rotor which is mounted at the center of an elastic shaft is analyzed. The equations of motion in dimensionless form are expressed as follows[1]:

$$\left.\begin{aligned}\ddot{\theta}_x + i_p\omega\dot{\theta}_y + c\,\dot{\theta}_x + D_{\theta x} + \theta_x + N_{\theta x} &= F\cos\omega t\\ \ddot{\theta}_y - i_p\omega\dot{\theta}_x + c\,\dot{\theta}_y + D_{\theta y} + \theta_y + N_{\theta y} &= F\sin\omega t\end{aligned}\right\} \qquad (1)$$

where θ_x and θ_y are the projections of an inclination angle θ of the rotor on the xz-and yz-planes in the rectangular coordinate system $O-xyz$ whose z-axis coincides with the bearing center line, $i_p\ (=I_p/I)$ is a ratio of polar and diametral moments of inertia, ω is the rotating speed, τ is the dynamic unbalance, $F=(1-i_p)\tau\omega^2$, c is the external damping coefficient, $D_{\theta x}$ and $D_{\theta y}$ are internal damping terms, $N_{\theta x}$ and $N_{\theta y}$ are nonlinear terms in restoring forces. The nonlinear terms are derived from the potential energy by[2]

$$\left.\begin{aligned}V_N(x,\ y) &= \sum_{\substack{i,j=0\\(i+j=3)}}^{3}\varepsilon_{ij}\,x^i y^j + \sum_{\substack{i,j=0\\(i+j=4)}}^{4}\beta_{ij}\,x^i y^j,\\ N_{\theta x} &= \frac{\partial V_N}{\partial\theta_x},\quad N_{\theta y}=\frac{\partial V_N}{\partial\theta_y}\end{aligned}\right\} \qquad (2)$$

Internal damping terms are expressed as follows[3].

$$D_{\theta x} = \frac{h\,\dot{\theta}_x'}{\sqrt{\theta_x'^2+\theta_y'^2}},\quad D_{\theta y} = \frac{h\,\dot{\theta}_y'}{\sqrt{\theta_x'^2+\theta_y'^2}} \qquad (3)$$

The frequency equation for a system with no damping and no nonlinearity is given by

$$G\ (p) \equiv 1 + i_p\omega\,p - p^2 = 0. \qquad (4)$$

Two roots $p=p_f$ and p_b change as shown in Fig.1. The cross point of curve p_f and straight line $p=+(1/2)\omega$ gives the resonance point of subharmonic oscillation of order 1/2 of forward whirling mode.

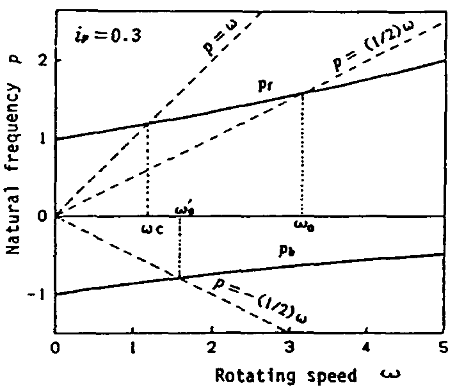

Fig.1 Campbell diagram

3. Theoretical Analysis

3.1 A self-excited system with no nonlinearity in restoring forces. ($N_{\theta x}=N_{\theta y}=0$)

The solutions are written asfollows.

$$\left.\begin{aligned}\theta_x &= Rp\cos(p_f t+\delta)+P\cos(\omega t+\beta)\\ \theta_y &= Rp\sin(p_f t+\delta)+P\sin(\omega t+\beta)\end{aligned}\right\} \qquad (5)$$

The amplitude $R=Rp_0$ of stationary solution is

$$\left.\begin{aligned}Rp_0 &= 0 && \text{for}\quad \omega<\omega c\\ Rp_0 &= h/(c\,p_f) && \text{for}\quad \omega>\omega c\end{aligned}\right\} \qquad (6)$$

The result is shown in Fig.2. A self-excited oscillation occurs in the wide region over the major critical speed.

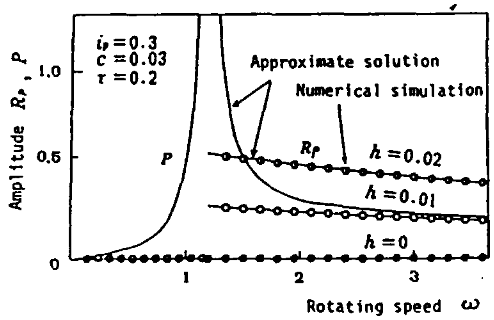

Fig.2 Resonance curves for a system with internal damping. ($N_{\theta x}=N_{\theta y}=0$)

3.2 A nonlinear system with no internal damping. ($D_{\theta x}=D_{\theta y}=0$)

The solutions of subharmonic oscillation at $\omega=\omega_0(=2p_f)$ is written as follows.

$$\left.\begin{aligned}\theta_x&=R\cos(\omega_f t+\delta)+P\cos(\omega t+\beta)\\ \theta_y&=R\sin(\omega_f t+\delta)+P\sin(\omega t+\beta)\end{aligned}\right\}\tag{7}$$

where $\omega_f=+(1/2)\omega$. The stationary amplitude R_0 for subharmonic oscillation is given by

$$R_0=0,\quad\text{and}\quad\{G_1+4\beta^{(0)}(R_0^2+2P^2)\}^2+(c\,\omega_f)^2=4\varepsilon^{(1)2}\tag{8}$$

where $G_1=G(\omega_f)$. A result is shown in Fig.3.

3.3 A system with internal damping and nonlinear spring characteristics.

(A) Entrainment In the neighborhood of the resonance point ω_0, a self-excited oscillation with frequency p_f disappears and only a subharmonic oscillation appears. The form of this entrained solution is the same as Eq.(7). Resonance curve for R_0 is given by

$$\{G_1+4\beta^{(0)}(R_0^2+2P^2)\}^2R_0^2+(-c\,\omega_f R_0+h)^2=4\varepsilon^{(1)2}P^2R_0^2\tag{9}$$

A result is shown in Fig.4. The solution $R_0\neq 0$ is separated from the trivial solution.

(B) Self-excited oscillation In the rotating speed regions which are beside the resonance point ω_0, a self-excited oscillation with frequency p_f appears together with a harmonic oscillation. Considering a derived frequency $q_f=\omega-p_f$, we suppose the solution as follows.

$$\left.\begin{aligned}\theta_x&=R\text{p}\cos(p_f t+\delta_p)+R\text{q}\cos(q_f t+\delta_q)\\ &\quad+P\cos(\omega t+\beta)\\ \theta_y&=R\text{p}\sin(p_f t+\delta_p)+R\text{q}\sin(q_f t+\delta_q)\\ &\quad+P\sin(\omega t+\beta)\end{aligned}\right\}\tag{10}$$

Results for the amplitude Rpo and the frequency p_f are shown in Fig.5. Some of these curves represent substantially the same curves as Fig. 4.

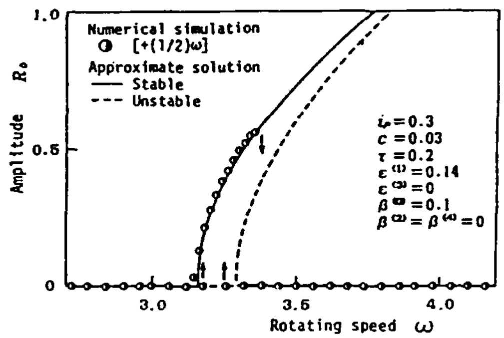

Fig.3 Resonance curves for a system with Nonlinear Spring Characteristics ($D_{\theta x}=D_{\theta y}=0$)

4. Experiments

Experiments were performed with an elastic shaft with a disc. The resonance curves at the resonance point of subharmonic oscillation are shown in Fig.6.

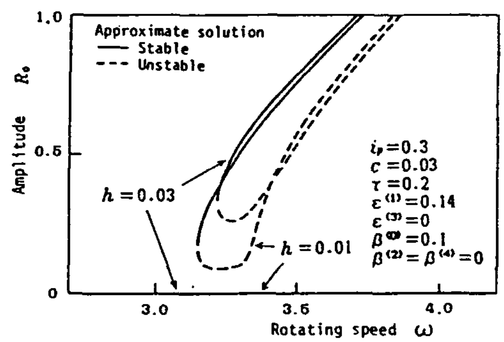

Fig.4 Entrained solution in a self-excited system with Nonlinear Spring Characteristics

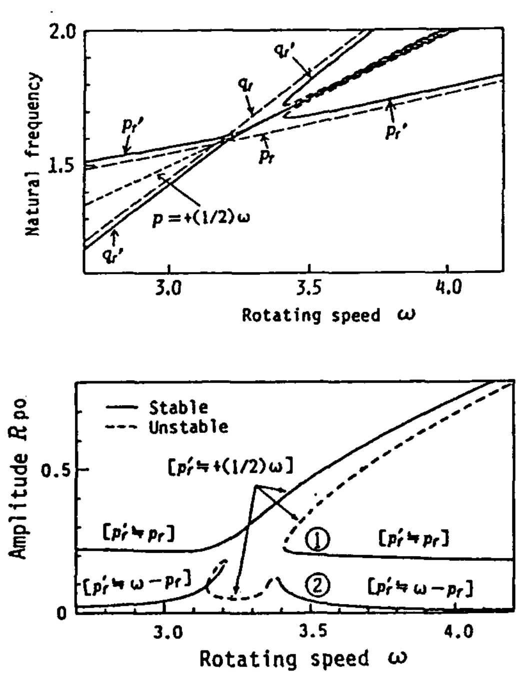

Fig.5 Resonance Curves for a self-excited system with Nonlinear Spring Characteristics

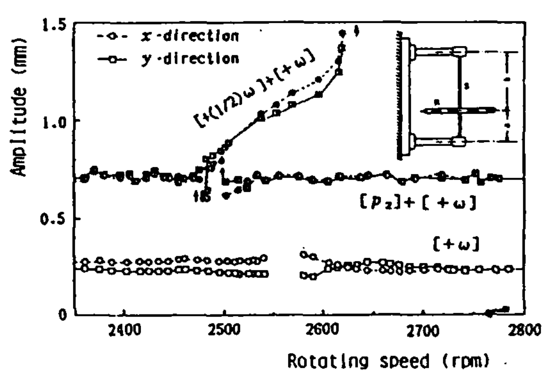

Fig.6 Experimental results

[literatures] (1)Yamamoto,T.et al.,Bull.JSME,Vol.18,No.123(1975),965. (2)Tondl,A.,Some Problems of Rotor Dynamics,(1965),Chap.1. (3)Yamamoto,T. and Ishida,Y.,Ing.-Arch.,46(1977),125.

Subharmonic Forced Traveling Waves in a Thin Perfect Circular Disk

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We study the subharmonic forced vibrations of a thin, isotropic circular disk. For finite amplitude motions, geometric nonlinearities due to midplane stretching result, and a variety of dynamic phenomena is observed. Because the disk is symmetric, pairs of modes corresponding to orthogonal nodal diameters exist. We use a Galerkin approximation to obtain the equations which govern the dynamic response of the system; the resulting equations are a system of coupled nonlinear ordinary differential equations with cubic nonlinearities. Furthermore, the assumed perfect symmetry of the disk leads to one–to–one internal resonances.

Subharmonic excitation is considered and the method of Multiple Scales is used to obtain the equations that govern the amplitudes and phases of two interacting "orthogonal" modes. The fixed points of these equations are then found numerically, and their stability is analyzed. Subharmonic forced traveling waves are identified, along with other standing–wave solutions. The traveling waves are created and destroyed through saddle–node bifurcations. The results are then verified by numerical integration of the equations of motion.

PERIODIC MOTION THROUGH A BIFURCATION

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A two mode model of ship motion describing nonlinear coupling between the pitch and roll modes shows an interesting bifurcation. The response to sinusoidal excitation has been analysed extensively by others (see, for example, the text, Nonlinear Oscillations by Nayfeh and Mook), and is well understood. As the wave excitation increases, the amplitude of the pitch mode increases linearly, with zero amplitude roll mode, until the pitch mode saturates; all further increases in input energy then feed into the roll mode via the nonlinear coupling. Depending on the degree of tuning between the excitation frequency and the two modal frequencies, rapid jumps from one locally stable solution to another can occur as the excitation amplitude increases.

An approximation to a narrow-band wave spectrum is made by using three sinusoids with closely spaced frequencies. Multiple time scaling is used to find the first order response. A restricted set of responses (perfect tuning, suitable initial conditions) is described by the equations:

$$\dot{a} + a + b = B_o + B_1 \cos \omega t$$

$$\dot{b} - ab = 0 .$$

a is associated with the pitch mode, b with the roll mode. The stationary, or unmodulated, solution with $B_1 = 0$ has a simple flip bifurcation at $B_o = 0$.

An obvious and exact solution has $b = 0$. In the stationary case this is stable only for $B_o < 0$. In the nonstationary case, the response shows effectively infinite penetration through the stationary bifurcation into the locally unstable region $(B_o + B_1 \cos \omega t) > 0$. The solution is shown to be globally stable for $B_o < 0$. Small perturbations either (i) decay immediately, (ii) grow immediately before decaying, or (iii) decay initially to extremely small values (appearing to be locally stable) before growing dramatically before their eventual decay. Analytic solutions are obtained that describe if and when the large growth occurs, and the overshoot. The solution is described by following the trajectories of psuedo-stable fixed points on the phase plane; solutions move towards and dwell close to saddle points before moving rapidly to a stable focus or node. The solution provides a graphic and simple example of how local stability theory is inadequate to describe the global response.

A second solution in the stationary case has $a = 0$, and is stable for $B_o > 0$. Corresponding nonstationary solutions are obtained approximately by (i) matched asymptotic expansions, and (ii) an ad-hoc approach at very low frequencies using the fact that the sum $(a+b) \approx B_o + B_1 \cos \omega t$. The approximate solution again estimates when the jump away from b very close to zero occurs. The minimum value of b can be extremely small; even so, this small value is critical in determining how long the solution dwells near a saddle point before moving rapidly towards a stable focus. A crude approximation is given also for the degree of overshoot. Numerical simulation is used to confirm the analytic solutions.

In the more general case, when the tuning is not perfect, the first order response is described by four first order equations rather than the two discussed above. The stationary solution appears to be well behaved. The nonstationary solution now involves more complicated jumps in amplitude. Numerical examples are given that show that associated with these jumps are additional bifurcations, including a sequence of period doubling bifurcations leading to chaos.

Work supported by the Natural Sciences and Engineering Research Council of Canada.

Formulation of a Vibration Control Law Based on Internal Resonance

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In recent years, there has been a considerable amount of research effort allotted to the development of techniques to control the vibrations in flexible systems. This is due, in part, but not limited to the need for lightweight robotic and structural technology for space applications. Two such examples are the Canadarm and the space station "Freedom".

There have been a variety of both passive and active vibration control strategies introduced over the past several years including active/passive joint configurations that utilize Coulomb friction to dissipate the energy from vibrating system, the use of piezoelectric materials as both actuators and sensors, the proof-mass actuator and proof mass damper, vibration suppression via tendon control and, most recently, vibration control via internal resonance.

Internal resonance is a dynamic phenomenon inherent to nonlinear systems. The particular state of internal resonance is dependent on the nature of the nonlinearities. That is, a system with quadratic nonlinearities can exhibit 2:1 internal resonance, a system with cubic nonlinearities can exhibit 3:1 internal resonance, etc. Mathematically, internal resonance is defined, primarily, via the *commensurability* of the natural frequencies of the system. That is, a nonlinear n-degree-of-freedom system can exhibit internal resonance if there exist constants, $(C_1, C_2, \cdots, C_n)$, such that $(C_1\Omega_1 + C_2\Omega_2 + \cdots + C_n\Omega_n \approx 0)$ where $(\Omega_1, \Omega_2, \cdots, \Omega_n)$ are the natural frequencies of the linear portion of the nonlinear system. Once a state of internal resonance is established, a transfer of energy between the modes of vibration transpires giving rise to a phase and *amplitude modulated* response.

In this paper, we propose a vibration control law to regulate the first mode oscillations of a flexible arm robot. The operation of the controller is contingent on modal coupling effects resulting from an internal resonance condition. To illustrate the feasibility of such a control law, we consider a very simple model. The plant is modelled by a second order differential equation which is representative of the first mode of oscillation of a single flexible robot arm, itself modelled as a cantilever beam. Attached to the base of the arm is a motor which is used to control the vibrations of the system. In order to establish an internal resonance condition, an additional second order, single-degree-of-freedom equation, termed the "dummy" equation, is introduced via software. The linear natural frequency of the dummy equation is defined such that the dummy equation and the plant equation form an internally resonant pair. We choose to exploit the effects of 2:1 internal resonance thus we incorporate *essential* quadratic nonlinear terms in both the dummy and plant equations to establish an artificial state of internal resonance via software. The nonlinear term associated with the plant equation becomes the equivalent torque required to establish the amplitude modulated motion that we desire - the result of a transfer of energy between modes of vibration. Therefore, the task at hand is to generate a torque profile that emulates the dissipation of energy once it has been transferred from the plant to the dummy system.

The controller design has been established on a *heuristic* basis. Through the selection of the magnitude of the coefficients of the nonlinear terms and the initial conditions imposed on the dummy equation, we can set the response rate and the steady state characteristics of the controlled system. The most novel result of the analysis is that the *control torque* is *unidirectional.* Under this control law, the torque profile always takes this form since the generated torque is proportional to the velocity of the second equation squared. This torque characteristic makes this particular control law ideal for systems using either thrusters or tendons as control actuators such as vibration control applications for *Large Space Structures* (LSS).

A similar structural configuration regulated under a PD (Proportional-Derivative) control law is compared to the proposed control scheme via simulation. If each controller is designed such that the controlled systems have similar rise times and settling times, the control torques are found to be approximately the same magnitude. The overwhelming advantage of the internal resonance controller is its unidirectional torque requirements as compared to the bidirectional torque requirements of the equivalent PD controller.

SESSION 8
APPLICATIONS TO MECHANICAL SYSTEMS
TUESDAY - 1030 - 1210
JUNE 9, 1992

CHAOTIC BEHAVIOR OF A PARAMETRIC NONLINEAR MECHANICAL SYSTEM

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ABSTRACT

The chaotic dynamics of a single degree of freedom nonlinear mechanical system under periodic parametric excitation is investigated numerically. The response to a principal parametric resonance of the same equation was previously considered by different autors [1,2]. In the present study, the amplitude and the frequency of the parametric forcing are taken as control parameters and are varied in the range [0.5-1.5] and [0.5-5.0] respectively.

Under these conditions the system exhibits a complex dynamics with a rich structure of coexisting attractors of distinct topological nature. Primary stable subharmonics up to period 12 have been seen numerically. All of these periodic stable solutions are born in Hopf bifurcations of the trivial solution of the equation or in saddle-node bifurcations[3,4].

A multitude of transitions to chaos were observed in various parameter ranges, frequently via the Feigenbaum route, but also type I and type III intermittency transitions to chaos were found. We also give numerical evidence that the system can follow an alternative route to chaos via intermittency from an equilibrium state to a chaotic one, which was not studied in the previous simulations of the dynamics of the system.

Besides the well known descriptions to identify the evolution of chaos (power spectral densities, Poincaré map, diagram of bifurcation ...), we have analysed the chaotic time series by using a transformation which gives a representation of the displacement or the velocity as a function of both time and frequency : the continuous wavelet transform [5]. The wavelet transform consists of expanding a time history s(t) over wavelets which are constructed from a single function by means of dilations 1/a and translations b. This tranformation can be seen as a mathematical microscope whose position is b and magnification is 1/a and whose optics are given by the choice of the specific wavelet. We have used a real wavelet called mexican hat, and a complex-valued one named the Morlet wavelet.

One of the aims of continuous wavelet transform is to provide an easily interpretable visual representation of signals, and a simple inspection of the wavelet transform allows us to differentiate two different chaotic time series. Since its implementation on a computer is not excessively time consuming and does not require large storage the wavelet transform provides a very efficient tool for analyzing chaotic dynamics.

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CHAOS IN THE UNBALANCE RESPONSE OF JOURNAL BEARINGS

R D BROWN *   P S ADDISON **   A H C CHAN **

*   Mech. Eng. Dept   HERIOT-WATT UNIVERSITY   EDINBURGH
**   Civil Eng. Dept   GLASGOW UNIVERSITY

Recently there has been an explosion of interest in non-linear systems in many areas of science and technology. It is now widely accepted that low dimensional systems which are deterministic can behave chaotically, essentially unpredictable. The classic example is based on the work of Lorenz in the early 60's which dealt with a simplified set of equations derived from weather prediction models. Duffing's equation has been shown to behave chaotically (UEDA 1973) and there are many other examples from a wide range of applications. The essential requirements for chaotic behaviour are non-linearity and at least three coupled 1st order systems.

An important mechanical component which has a strong non-linearity is the fluid film bearing. This is particularly true of journal bearings which are widely used in turbo-machinery. A popular analytical model can be derived from Reynolds lubrication equation by neglecting the circumferential pressure gradient contribution to flow. The resulting PDE is often used to provide a compact set of equations to calculate load, friction losses and flow. The equation can be linearised to yield stiffness and damping coefficients for rotor response and stability investigations. When these coefficients are used to calculate dynamic response to mass unbalance elliptical orbital motion is obtained. However non-linear solutions yield distorted orbital patterns especially when the journal displacement approaches the bearing clearance. Holmes investigated large amplitude response and published a number of limit cycles. Ettles (1978) et al demonstrated, using numerical integration methods, that aperiodic behaviour was possible for low levels of unbalance.

Chaotic behaviour is ~~also~~ likely when the magnitude of the unbalance force exceeds the gravitational force. In this case the unbalance force is intermittently counteracted by gravity. This effect is magnified by the small load instability of plain journal bearings. If the unbalance magnitude is increased then the rotating force dominates and hence the behaviour is moderated. The transition from regular limit cycle response to the chaotic regime , as the unbalance level is increased, is revealed using frequency spectra and Poincare sections of phase-plane data. Further consideration of dynamic/static force ratio show the present large unbalance chaotic case can be shown to lie on the low unbalance aperiodic boundary given by Ettles.

More sophisticated methods of demonstrating chaos in a particular system are based on establishing the fractal dimension of the strange attractor revealed by Poincare sectioning. One such method , the Grassberger-Procaccia algorithm, is based on the construction of a pseudo-attractor of N dimensions using a method of time delays and data from a single co-ordinate. The correlation measurement obtained is known to give a lower bound on the fractal dimension. For the bearing model considered the attractor dimension is established at 2.15 for embedding dimensions of up to 12. This indicates that the dynamics of the journal bearing exist on a finite, low-dimensional attractor, a typical result for chaotic behaviour in dissipative systems with only a few degrees of freedom.

NONLINEAR RESPONSE OF A CLASS OF ENGINE MOUNTS

by

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EAST LANSING MICHIGAN 48824-1226

The automobile engine provides a formidable vibration isolation problem. Due to its nature, it is the cause of forces and acoustic noise which are transmitted to the rest of the automobile. These disturbances range in frequency from 6 - 15 Hz (engine rigid body modes) to 20 - 40 Hz (typical idle speeds) to higher frequencies due to engine combustion. The engine also must be protected from gross displacements which may occur due to road excitation, turning, stopping, etc. These conflicting vibration isolation requirements, low stiffness for idle vibration isolation and high stiffness and damping for motion control, has led to the development of the hydraulic engine mount as a replacement for the more standard elastomeric type.

This talk shall present a brief description of the physical operation of this class of mount, drawing attention to the source of many gross nonlinearities. For example, the flow of fluid through specially design tracks within the mount. The resulting behavior is further complicated by the addition of a mechanical decoupler which creates an amplitude dependent alternative path for the fluid to flow through [1,2,3].

The results of an ongoing study into the complex nonlinear behavior of such mounts shall be presented. Results from experiments conducted on mounts currently being used in the upper-end of the car market will highlight the shortcomings of the linear modeling and testing techniques that are currently used by industry. The response of the mount to a simple sinusoidal displacement will be discussed and attention focused on more realistic mathematical models which can be used to predict the mounts behavior. The existence of chaotic responses shall be demonstrated along with a variety of a number of other unusual phenomena. As is widely appreciated, engine vibrations contain a number of harmonics. With this in mind, the response of the mount to multi-harmonic input will also be discussed.

Finally, having presented an overview of the nonlinear types of response that are observed, attention will be focused on ways in which the presence of the nonlinearities can be used to enhance the overall performance of the mount.

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1. "Hydraulic Engine Mount Characteristics," P.E. Corcoran and G. Ticks, SAE Paper 840407, Society of Automotive Engineers, Warrendale, PA.

2. "Understanding Hydraulic Mounts for Improved Vehicle Noise, Vibration and Ride Qualities," W.C. Flower, SAE Paper 850975, Society of Automotive Engineers, Warrendale, PA.

3. "Linear Analysis of an Automotive Hydro-Mechanical Mount with Emphasis on Decoupler Characteristics," R. Singh, G. Kim, P.V. Ravindra, to appear, Journal of Sound and Vibration.

DYNAMICS OF A FOUR WHEEL STEER VEHICLE

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Division of Engineering
The University of Texas at San Antonio

Abstract

The present work develops a nonlinear analysis of the response of an all wheel steer vehicle. Maneuvers in response to steer angle inputs can be accurately described by neglecting the effect of the suspension and using a simple two-degree-of-freedom model including lateral displacements and yaw. The lateral and directional dynamics in this model are dominated by the tire side force properties which exhibit clear nonlinear characteristics. The governing equations are derived, and the analytic and numerical solutions obtained show that the traditional simplifications used in control analysis lead to considerable errors and underestimation of potentially dangerous responses.

A TRANSMISSION MERIT PARAMETER FOR PLANAR MECHANISMS

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ABSTRACT

In mechanism design, the pressure angle or transmission angle is conventionally used as an indicator of the quality of motion and force transmission for a number of planar mechanisms including linkages and cam mechanisms. Perhaps the earliest in-depth study of the transmission quality is the work done by Hartenberg and Denavid [1]. They differentiated the displacement equations for a four-bar function generator, and derived an expression for the transmission angle and the output sensitivity to link length perturbations. They also showed that a mechanism with a poor transmission angle is subject to large mechanical errors. A crucial development in this area is the concept of transmission index introduced by Sutherland and Roth [2], which extended the concept of transmission angle to any spatial linkages. This index is shown to be related to the possible mechanical error in a mechanism. By using these ideas, much work has been done for simple planar, spatial and spherical function and path generators, which aimed to maximize the transmission quality of the mechanisms. However, the generalization of these methods to more complex mechanisms has not been addressed. The key question seem to be what quantity plays the same role for multi-loop and combined mechanisms as the transmission index for the simple mechanisms.

A Transmission Merit Parameter (TMP) is defined in this paper which comprehensively reflects the transmission quality and the output sensitivity of a mechanism to dimensional disturbance. The TMP is derived from direct differentiating of the system loop closure equations, and it provide a mapping from the system input(s) to the system output(s). In contrast to the transmission index, TMP is a function of both character angle(s) and the link lengths.

It is shown that the Transmission Merit Parameter is an extension of the conventional transmission angle to multi-loop and combined mechanisms, since the character angle in this parameter is simply the conventional transmission angle when the mechanism is a simple one such as a four-bar linkage. It is also shown that the idea of TMP is suitable for mechanisms with multi-degrees of freedom, and that without a measure such as TMP, representation of the transmission quality for combined and multi-loop mechanisms would be a difficult task. It is suggested in this paper that the TMP also be used for dimensional synthesis or optimizing mechanisms performing certain tasks.

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[1] Hartenberg, R.S., and Denavit, J., **Kinematic Synthesis of Linkages**, McGraw-Hill, 1964.

[2] Sutherland, G., and Roth, B., "A Transmission Index for Spatial Mechanisms," Transaction of the ASME Journal of Engineering for Industry, pp. 589, 1973.

SESSION 9
RANDOM VIBRATIONS
TUESDAY - 1330 - 1510
JUNE 9, 1992

Applications in Nonlinear Soil/Structure Interaction

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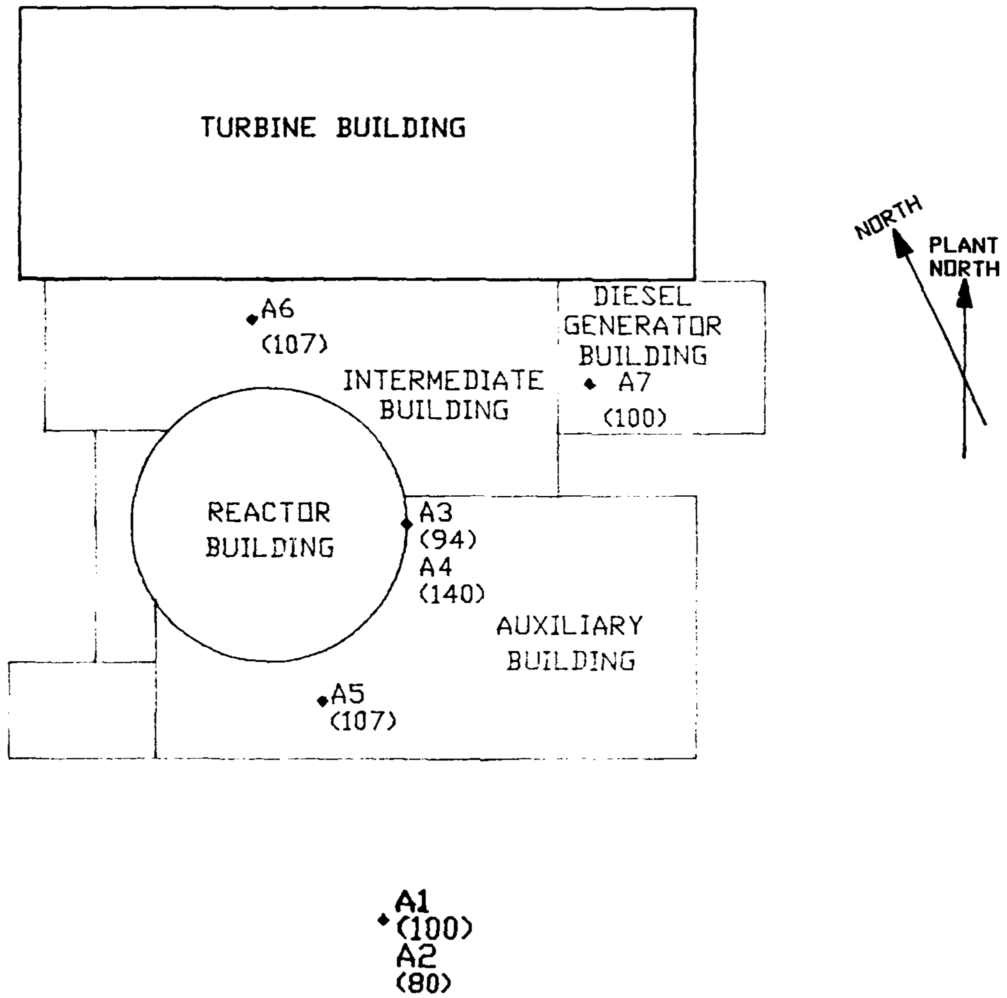
Two eastern U.S. nuclear power plants, V.C. Summer in South Carolina and Perry in Ohio, went through extensive licensing effort to obtain fuel load licenses when recorded earthquakes on sites exceeded the original design basis. On December 1989, Krsko nuclear power plant, in the Republic of Slovenia, Yugoslavia, experienced a short-duration high-frequency high-amplitude earthquake that triggered plant seismic instrumentation. Acceleration records were obtained inside the buildings and a nearby free-field recording station. Accelerometers were installed on the base mats of the Reactor and Diesel Generator Buildings and one free-field accelerometer was installed in a small shelter located 50 m south of the Reactor Building. Each of these accelerometers provides translational motions in the East-West, North-South and Vertical plant directions. Figure 1 schematically shows the plan of Krsko nuclear power plant, locations of earthquake instrumentation and recorded peak ground accelerations.

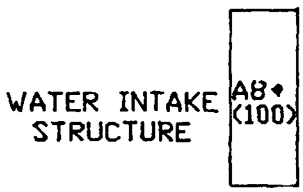
This paper presents the common characteristics of the Krsko earthquake and Eastern U.S. Type earthquakes, assesses damage potential, assures structural integrity of these safety-related power plants and describes results of an analytical investigation of how the records of the Krsko earthquake may be influenced by soil/structure interaction. This investigation is based on soil data, that was obtained from the pre-construction geotechnical investigations at the site including few parametric studies to account for uncertainties in the soil properties. These consist of variations in the shear and compressional wave velocities and variations in the seismic wave environment in the form of arbitrarily oriented body waves or Rayleigh surface waves.

The analysis takes into account nonlinearity of the soil material, radiation and hysteretic damping, viscoelastic halfspace, ground-water table level, structural embedment, soil/structure interaction, and structure/structure interaction. The analysis is based on state-of-the-art computer software, elaborate analysis techniques and simpler engineering approximations.

Results of analysis show clear evidence of strong soil/structure interaction, nonlinear softening of the soil material and encouraging qualitative and quantitative agreement with the recorded measurements. The structural response motions display high rocking mode participation due to the high-frquency content of the free-field input motion.

N. E. KRSKO
DECEMBER 1989 EARTHQUAKE
PEAK RECORDED ACCELERATIONS





ACCELERO-METER	LOCATION	PEAK ACCELERATION N-S	VERT	E-W
A1	FREE-FIELD EL. 100	0.45	0.13	0.53
A2	FREE-FIELD EL. 80			
A3	R.B. BLDG. EL. 94	0.08	0.04	0.05
A4	R.B. BLDG. EL. 140	0.06	0.07	0.06
A5	AUX. BLDG. EL. 107			
A6	INTER. BLDG. EL. 107			
A7	D.G. BLDG. EL. 100	0.14	0.16	0.19
A8	INTAKE STRUCT. EL. 100	0.10	0.06	0.06

Chaotic Motion and Stochastic Exctitaion.

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°°Professor, Department of Structural Mechanics,
University of Pavia - Italy.

The behavior of nonlinear systems that are chaotic and/or stochastic is investigated. Their initial state can only be specified with finite precision. This uncertainty leads to the incapacity of predicting the long-time motion of the system.

Attention is focused on the chaotic behavior of a nonlinear dynamical SDOF system under harmonic and stochastic excitation. The behavior of this oscillator was characterized in past studies by techniques associated with power spectra, Poincare' sections, Lyapunov exponents, capacity and information dimensions and probability densities.

The Kolmogorov's entropy K determines the average time over which the state of a system, displaying deterministic chaos, can be predicted. After this time, one can only make statistical predictions and the numerical calculation of single trajectories loses its meaning to describe the system behavior. In higher dimensional systems, K measures the average deformation of a cell in phase space and becomes equal to the integral over phase space of the sum of positive Lyapunov exponents.

Since a direct computation of K is presently unpracticable, one looks at the evolution of a set of many identical oscillators with close initial conditions . One computes the evolution in time of the entropy of the set of systems and derives informations on K from the behavior of this function.

The result is an algorithmic process able to identify the nature (chaotic rather than stochastic) of the system behavior.

Horizontal-Vertical Response Spectra for El Centro Earthquake

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Abstract

A single-degree-of-freedom structure subjected to horizontal and vertical earthquake ground acceleration is considered. The corrected accelerograms of El Centro 1940 earthquake are used. For a range of values of parameters, maximum horizontal responses are evaluated. Particular attention is given to the amplification effects of the vertical ground acceleration. Combined horizontal-vertical response spectra curves are developed. The results are compared with some peak response estimates and reasonable agreement is observed. A procedure for developing site-dependent smooth spectra is also outlined.

Stochastic Response of a Parametrically Excited Buckled Beam Under Wide-Band Random Excitation

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The stochastic response of a simply-supported buckled beam to a wide-band random excitation is investigated using analog-computer simulations. The random signals of the excitation and response are processed to determine their mean squares, autocorrelation and cross-correlation functions, phase portraits, power and cross-spectral densities, cumulative distribution functions, and probability density functions. The analog-computer results are qualitatively compared with those predicted by solving the moment equations obtained from the the Fokker-Planck-Komologorov (FPK) equation using both a Gaussian and a non-Gaussian closure scheme. The simulation results show that the system response is a narrow-band process. The measured probability density functions of the response suggest that the Gaussian closure scheme is sufficient for relatively low levels of excitation.

LYAPUNOV EXPONENTS AND INFORMATION DIMENSIONS OF NONLINEAR SYSTEMS UNDER DETERMINISTIC AND STOCHASTIC EXCITATIONS

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Abstract

The important concept of Lyapunov exponent has emerged in many fields in the last decade. For examples, it can be found in the fields of random matrices and random maps, nonlinear stochastic systems, stochastic flows on manifolds, chaos and phase transitions. It plays a crucial role in the determination of chaotic motions in nonlinear systems. Strategies for its numerical computation of general single and multi-degree-of-freedom (MDOF) nonlinear systems under deterministic excitations are available in the literature. However, for nonlinear systems under stochastic excitations techniques available for the determination of Lyapunov exponents are very limited. They are essentially confined to single degree-of-freedom (S DOF) systems with small nonlinearities and small force. For two DOF systems with small nonlinearities and small forces it is restricted to non-resonant cases.

While nonlinear systems with small and large deterministic forces, and nonlinear systems with small stochastic excitations have many engineering applications, there are nonlinear dynamic engineering systems excited by very large deterministic and stochastic forces over a relatively long duration of time. For examples, the thrust from the propulsion system of a rocket or a jet engine of an aircraft and the force exerted on the wings of an aeroplane by atmospheric turbulence. To be able to design the above systems more reliably and economically it is imperative that the question of stability and the nature of motion, be it ordered or chaotic, be addressed qualitatively and quantitatively. Furthermore, it is also of interest to know the consequence of a nonlinear system disturbed by a large stationary force. Thus, there is a need to develop analytical techniques and numerical strategies for the determination of Lyapunov exponents of nonlinear systems under large stationary forces.

This investigation is concerned with the development of a numerical strategy for the computation of Lyapunov exponents of SDOF and MDOF nonlinear systems under small or large stochastic excitations. For comparison purpose, Lyapunov exponents of nonlinear systems under large deterministic forces are obtained by using the strategy and computer program of Wolf, Swift, Swinney and Vasano (1985). In addition, the concept of information dimension for these systems is applied. For illustration of the use of the proposed strategy the results of a Duffing oscillator are obtained. While in the case of deterministic excitations the Lyapunov exponents and information dimensions, over an entire range of magnitudes of the force studied, are very irregular and they show ranges of ordered motions among chaotic ones, in the case of stochastic excitations the ensemble averages of Lyapunov exponents and information dimensions based on the ensemble averages of Lyapunov exponents, over an entire range of spectral densities, are smooth and chaotic. In the latter case there is an optimal information dimension over the entire range of stochastic force magnitudes.

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SESSION 10
ANALYTICAL METHODS II
TUESDAY - 1530 - 1710
JUNE 9, 1992

The Nonstationary Period Doubling Route to Chaos

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Abstract

In this study, a computational analysis has been initiated with a view of determining the effects of nonstationary processes of two time dependent control parameters on period doubling bifurcation cascade—a typical route to chaos. The driven, damped, with negative (softening) nonlinear coefficient Duffing oscillator with forcing amplitude and frequency varying linearly with time (constant sweep) has been employed. The Duffing oscillator is widely used to mathematically model a variety of engineering and physical systems. The stationary $2T, 4T \ldots \chi$ bifurcation regions and their boundaries have been determined. Nonstationary regimes used consisted of (1) transverse transitions through the bifurcation regions with $f(t) = f_0 \pm \alpha_f t$, $\Omega = \text{const.}$ and (2) transitions along the lower boundary of the $2T$ region or L-line for $f(t) = f_0 \pm \alpha t$; $\Omega(t) = \Omega_0 \pm \alpha t$.

New revealing, at times puzzling and always complex behavior of nonstationary period doubling bifurcations has been uncovered. The study also establishes an agenda for nonlinear dynamics and chaos in nonstationary regimes.

Prediction of escape from a potential well under harmonic excitation

by

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and

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Abstract

The nonlinear systems considered here are initially at rest in a stable (reference) equilibrium state and then are subjected to harmonic excitation. For sufficiently small forcing amplitudes, the motions remain in the neighborhood of the reference state. At a critical value of the excitation, these systems exhibit an abrupt jump in their response. Our objective is the determination of this critical force by an approximate technique. There are basically two types of mechanisms involved here. First, the jump may correspond to the loss of stability of a steady-state cycle (e.g., the classic jump phenomenon in nonlinear resonance) or it may be triggered by a sequence of period doubling bifurcations. Second, the jump may be a direct result of transient behavior.

For structures such as arches or shells, this jump in response may correspond to "snap-through" instability, in which the curvature changes sign and part or all of the structure may invert [1]. For other structures, it may be associated with "overturning" under wind or earthquake loading. For ships, the jump may be associated with capsize [2]. If the system is conservative when damping and forcing effects are not included, the potential energy has a minimum at the reference state, which lies in a "well". A "potential barrier" must be

overcome for the jump in response to occur, and hence this behavior is often called "escape from a potential well" [3].

The approximate method used here has three features. The first is the determination of an approximate steady-state motion. This will be accomplished with the harmonic-balance technique, and the solution will be assumed to have the same frequency as the excitation. The second feature is the determination of the unstable equilibrium states of the undamped, unloaded system. If only one such state exists, or several states but all with the same potential energy, the potential energy at these states (relative to the reference state) is denoted V_s. Otherwise, V_s will be the potential energy of the "nearest" state, which is reached first as level surfaces of constant potential energy are expanded about the reference state. Thirdly, the maximum total energy, E_{max}, during the approximate steady-state motion is determined. The approximate critical forcing amplitude for escape is taken to be the lowest value for which $E_{max} > V_s$ (i.e., the system has enough energy to overcome the potential barrier). In some cases, a safety factor ρ is included and the condition $E_{max} > \rho V_s$ is used, with $\rho = 1$ when there is no safety factor. The method is applied to a number of examples, and comparison is made with the critical loads obtained by direct numerical integration of the equations of motion [1].

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1 Assessing and quantifying the engineering integrity of nonlinear vibrating systems in terms of basins of attraction

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Abstract

Complex phenomena including chaotic oscillations, dangerous bifurcations, co-existing solutions and fractal basins are typical of nonlinear dynamical systems. However when simplicity can be derived out of such complexity, a nonlinear analysis can have important engineering relevance.

Unlike linear systems where all initial conditions lead to one type of motion; be it to an equilibrium point or to a harmonic oscillation, nonlinear systems can exhibit a rich and complex variety of competing steady state solutions (attractors). All initial conditions that generate trajectories that tend towards an attractor define its phase-space *basin* or *domain of attraction*. Indeed, there has been much interest in basins of attraction, and how they undergo changes and metamorphoses. Under the variation of a control parameter, such as the forcing frequency for example, as the attractors move and bifurcate, the basins also undergo corresponding changes and metamorphoses.

For typical nonlinear systems, which include systems subjected to external, parametric or other types of excitation, complex phenomena including subharmonic and chaotic oscillations, dangerous bifurcations, co-existing small and large amplitude oscillations and fractal basins can be observed.

In assessing the engineering significance of such complex chaotic phenomena, more attention should be paid to basins of attraction and their basin boundaries rather than the intricate patterns of the bifurcating steady states. This is particularly important for engineering systems operating under transient conditions in noisy or ill-defined environments, as regular excitation will manifestly never persist long enough for transients to have decayed as to allow for a steady state analysis. Furthermore, since the initial conditions at the beginning of the excitation may vary widely and indeed are unknown, we must look at *all* possible transient motions, rather than focus on just the one steady-state motion.

As recent studies have shown such by adopting such an approach there can exist a rapid and dramatic erosion of the basin long before the final steady state solution loses its stability. This

conclusion is reinforced by the fact that basin boundaries can become *fractal*, adding a new degree of uncertainty in the response.

In this paper we present, for typical types of nonlinear systems, basin erosion studies, that exhbit this type of behaviour. For systems operating in essentially transient conditions, we quantify the global integrity or stability of the system in terms of the safe basin of attraction. We also assess this behaviour for systems operating under essentially steady-state conditions. The local stability, (but not in the infinitesimal sense), of the steady state, say to an impact loading, can be quantified in terms of the local basin structure.

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Influence and Equivalence of Different Ship Roll-Damping Models through a Melnikov Analysis

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In this study, we examine the influence of roll damping on the rolling motions of an unbiased ship in regular beam seas in this study. In particular, we are interested in motions that could potentially cause dynamic capsizing. Assuming that the roll motion is uncoupled from the other motions and that the ship does not possess any forward speed, we model the ship as a single-degree-of-freedom system. A general damping model is considered and a Melnikov analysis is conducted to assess the influence of the different damping terms. Using phase-plane concepts, we obtain simple expressions for what we call the Melnikov damping coefficients.

As an application, we consider the equivalence of linear-plus-quadratic and linear-plus-cubic damping models, systematically investigated by Dalzell [1]. Dalzell compiled experimental records from several sallying experiments and showed that with properly chosen damping coefficients both damping models yield identical roll-extinction curves. Hence, Dalzell concluded that, a linear-plus-cubic damping (LPCD) model could be used to model ship-roll damping. For mathematical reasons, the LPCD model is preferable over the LPQD model because the cubic damping term is infinitely differentiable while the quadratic damping term is only once differentiable. First, we follow Dalzell in choosing the damping coefficients such that the roll-extinction curves are identical for both damping models. This choice requires that the two damping models be close in a least-squares sense over some domain of matching that is fixed a priori and assures that the two damping models will yield identical free-oscillation behavior. However it is not known if the two chosen damping models will yield identical forced-oscillation behavior. Balachandran, Bikdash, and Nayfeh [2] examined the frequency-response curves, the Melnikov predictions for the maximum safe wave slope, and the basins of safe and capsize regions and concluded that the two damping models yield different transient and steady-state forced-oscillation behavior.

It is of interest to examine under what conditions the LPCD and LPQD models can yield identical forced-oscillation behavior besides identical free-oscillation behavior. Here, we explore if it is possible to choose the domain of the least-squares approximation such that the two models yield the same

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Melnikov predictions. From a Melnikov analysis, we infer that the influence of the different damping terms on the Melnikov function is simple and additive. Further, we derive a condition under which the two models can be made to yield the same Melnikov predictions and show that the resulting models have the same frequency-response curves and very similar basins of safe and capsize regions. Thus, with this judicious choice of the matching domain, the two models yield the same free-decay response, and the same steady-state, and similar transient forced responses. A detailed presentation of the results will be made in the full paper.

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NONLINEAR AND CHAOTIC OSCILLATIONS OF A CONSTRAINED CANTILEVERED PIPE CONVEYING FLUID: A FULL NONLINEAR ANALYSIS

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In this paper, the planar dynamics of a constrained cantilevered pipe conveying fluid is examined numerically by considering the full nonlinear equations of motion. The linear and nonlinear dynamics of pipes conveying fluid has been studied quite extensively, both theoretically and experimentally, over the past thirty years. In a recent survey of the subject [1], over two hundred papers on various aspects of the problem were reviewed. It was shown that the pipe conveying fluid has become a premier paradigm in dynamics.

The present study completes the circle of several studies by examining the effect of other than restraint–related nonlinearities in the equations of motion on the dynamics of the system. Indeed, Païdoussis & Moon [2] studied the dynamics of a cantilevered pipe constrained by motion limiting restraints, and showed, both theoretically and experimentally, that chaotic oscillations occur at sufficiently high flow velocities. In their study, three principal idealizations were introduced: (i) the linearized equations of motion were utilized, apart from the nonlinear impact force term; (ii) a two–mode Galerkin discretization of the equations of motion was used for analysis; (iii) the trilinear spring was idealized by a cubic one. They were able to prove that the chaotic–looking oscillations, after a period–doubling cascade, were indeed chaotic. Despite these idealizations, the correspondence between theoretical and experimental results was remarkably close qualitatively; but, quantitatively, there remained a fair margin for possible improvement. This was partially done by Païdoussis, Li & Rand [3].

Here, *the full nonlinear equations of motion* [4] *and a refined trilinear spring model* for the impact constraints are utilized, the number of degrees of freedom N is increased and the results are compared to those in the foregoing studies. With the aid of modern numerical techniques, involving the construction of phase portraits, bifurcation diagrams and power spectra and the determination of Lyapunov exponents, some rather interesting and unexpected results have been obtained.

The inherent nonlinearities reduce the amplitude of motion of the pipe, rendering the system much more stable, from a physical as well as from a numerical point of view. Even for a two-mode model ($N = 2$), very good *qualitative* and *quantitative* agreement is obtained between theory and experiments. For example, after the region of chaos, it is found that the system becomes unstable by divergence, which has been observed experimentally. For $N = 3$, some unusual results are obtained for some parameters; beyond the pithfork bifurcation, rather than getting the cascade of period-doubling bifurcation, the amplitudes of the oscillations decrease: period-bubling is found to exist. For higher values of N, the results are again very close to what is expected. For $N = 4$, it is shown that as compared to the nonlinearity due to the motion constraint, the effects of other nonlinearities on the system dynamics are rather small.

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SESSION 11
ANALYTICAL AND SYMBOLIC METHODS
WEDNESDAY - 0830 - 1010
JUNE 10, 1992

On The Nonlinear Parametric Excitation Problems Of A One And A Half Degrees Of Freedom System

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Abstract

In this paper, the nonlinear parametric excitetion problems of a one and a half degrees of freedom system describeng the dynamics problems of a electromagnetic mechanical coupled system in the energy industry are studied.

All connected phase isolated busbar which consists of conductor, insulator, elastic supporting, and aluminium outer shell is used for transmission of large electric current from large generator. As the shield mechanism of aluminium outer shell, the conductor is subjected to motion electrodynamical force which depends on the displacements and the velocities of points in the busbar. Thus the calculation of the short circuit load and the analysis of dynamical response of the conductor–insulator system will not be proceeded independently and so they form a electromagnetic mechanical coupled problem. Since the mathematic model is complicate and it is necessary to find out the new dynamical behaviors of this coupled system for engineering, this study is then raised.

In the case of elastic supporting, the coupled system can be described by the third order nonlinear ordinate differential equation with variable coefficients. (The "third order" corresponds the displacement and velocity of conductor and magnetic field.)

$$\frac{d^3u}{dt^3} + b_2\frac{d^2u}{dt^2} + b_1\frac{du}{dt} + b_o u + 3cu^2\frac{du}{dt} + b_2 cu^3 = f \tag{1}$$

$$b_2 = \alpha - \omega sin\omega t, \quad b_1 = \mu - 2\beta cos\omega t + \frac{\beta}{2}cos2\omega t, \quad b_0 = \delta - 2\varepsilon\omega sin\omega t,$$

$$f = F(1 - cos\omega t)(e^{-\mu t} - e^{\sigma t})$$

where u is the displacement of the conductor, ω is the frequency of current

This is a dynamics problem of a nonlinear oscillation system under combined parametric and forcing excitation. Here, we study the following dynamics topics of this special system.

Instability of linearized unforcing system of the original system

According to Floquet theory, the boundaries of the instability regions of the linearized unforcing parametric excitation system of system (1) (c = 0, F = 0)

are determined by the periodic solutious of this system. For finding the periodic solutions there are some differences between this systm and the typical second order Mathieu equation that first the equation of the linearized unforcing system includes not one but three time variable coefficients and the number of the parameters is more than two. Second corresponding the half degree of freedom, there is no appearing of periodic solution. For using asymtotic method, we set the parameters as follows

$$\alpha=\varepsilon\alpha_1, \quad \mu=\omega^2+\eta, \quad \eta=\varepsilon\mu, \quad \beta=\varepsilon\beta_1, \quad \delta=\delta_0+\varepsilon\delta_1 \tag{2}$$

where ε is small. Using strained parameter method, we obtained several results; one of them is that the boundary of the instability region corresponding a first periodic solution of the linearized unforcing system is a three dimensional surface in the four dimensional parametric space (η, β, δ, ε):

$$-\eta^2\omega^2+\frac{1}{16}\varepsilon\beta\omega^2-\delta^2=0 \tag{3}$$

Response and Instability of nonlinear unforcing system

In the original electromagnetic mechanical coupled system (1), there are two nonlinear terms $3cu^2du/dt$ and b_2cu^3, one of them, b_2cu^3, possesses time variable coefficient b_2c. This is also a new case of parametric excitation problem. Here we study the unforcing system of (1) (F = 0). Using the asymtotic method of Krylov, Bogoliubov and Mitropolsky, the approximative equations of the series solution, and the corresponding amplitude and phase angle differential equation are derived. Among them, the first approximaive equations of amplitude and phase angle are

$$\frac{da}{dt}=\left(\frac{\alpha}{2}-\frac{\delta}{2\omega^2}+\frac{3\beta}{16\omega}\sin2\theta\right)a, \quad \frac{d\theta}{dt}=-\frac{1}{2\omega}\left(\mu-\frac{\beta}{2}+\frac{3}{4}\gamma a^2+\frac{3}{4}\beta\sin^2\theta\right) \tag{4}$$

And the conditions of the existence of the stationary response in the first instability region are obtained:

$$\frac{3}{8}\beta\omega\geqslant\left|-\alpha\omega^2+\delta\right|, \quad -\eta+\frac{\beta}{8}+\sqrt{\left(\frac{3}{8}\beta\omega\right)^2-(-\alpha\omega^2+\delta)^2}\geqslant0 \tag{5}$$

This is the case of limit cycle of original system. Besides, the responses and instablity of the various special cases and the bifurcations of the responses are also discassed.

The short circuit dynamical respouse of the coupledsystom

By the numerical analysis on this stiffness equation, a number of results available forengineering are obtained.

The Symbolical Analysis of Nonlinear Systems of Differential Equations

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The aim of this work is the localization of the set of the chain recurrent points. The method proposed here permits one to check the condition of transversality on the chain recurrent trajectories.

Let us consider a system of differential equations

$$\dot{x}=f(x), \qquad (1)$$

where $x \in M$-manifold and f is C^1 vector field. Let $X(t,x)$ the solution of this system and $X(x)=X(t_0,x)$, $t_0>0$. An infinite sequence $\{x_k\}$ is ε-trajectory, if for any k the distance $\rho(X(x_k),x_{k+1})<\varepsilon$. If in this conditions the sequence $\{x_k\}$ is periodic, it's called the periodic ε-trajectory. A point x is called chain recurrent, if for any positive ε there exists the periodic ε-trajectory, which passes through the point x.

Let's denote by Q the set of the chain recurrent points. This set is invariant, closed and contains singular trajectories: periodic trajectories, almost-periodic trajectories, recurrent trajectories and so on.

The Construction of the Symbolic Image. Let a collection of close sells $\Sigma=\{M_1,\dots,M_n\}$ be the finite covering of the manifold M. Let us build the covering Σ_i of the image $X(M_i)$ by cells from Σ, putting $\Sigma_i = \{ M_j\colon M_j \cap X(M_i) \neq \emptyset \}$. Let us construct the oriented graph Γ , associating to each cell M_i the vertex i . The vertexes i and j are connected by the oriented edge (i,j) only in case if the cell M_j is included in the covering Σ_i. Such the graph Γ is called symbolic image of the diffeomorphism X and dynamical system (1). It can be said, that the symbolic image is a finite approximation of the diffeomorphism X. The vertex i of the symbolic image Γ is called recurrent, if there exists closed path on Γ passing through i. Let's denote by $P(d) = \cup M_i \{ M_i$: i - the recurrent vertex $\}$, where d is the largest diameter of the cells M_i.

Theorem 1. The set P(d) is the neighborhood of the set of the chain recurrent points, moreover this neighborhood is sufficiently small, if the maximum diameter d is small enough.

Theorem 2. The set of chain recurrent points $Q= \bigcap_{d>0} P(d)$ for all positive d.

Theorem 3. There exists the algorithm of the construction of the imbedded neighborhoods of Q $P_0 \supset P_1 \supset P_2 \supset \ldots$, such that $\lim_{k \to \infty} P_k = Q$.

<u>The construction of a linear extension over the symbolical image.</u> Using the differential ∂X, for each oriented edge (i.j) a linear map A(i,j) is associated. It may be said, that the linear maps A(i,j) are a finite approximation of the differential ∂X. The oriented graph Γ together with the linear maps A(i,j) is called linear extension over the symbolical image.

Theorem 4. If the periodic path ω on Γ is hyperbolic and the largest diameter d is sufficiently small, then the system (1) has periodical trajectory and there exists the canonical algorithm of construction of this periodical trajectory.

We created the computer programs for realization of the algorithms, named here.

Nonlinear Vibration of a Flexible Connecting Rod

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An analytical and computer simulation investigation of the dynamic behavior associated with the flexible connecting rod of an otherwise rigid, in line, planar slider-crank mechanism is presented. The main goal is to analyze the flexural response of the elastic connecting rod and determine how this response depends on the system parameters. Moreover, this work emphasizes nonlinear aspects of the dynamic response and its associated stability.

The flexural vibration of the connecting rod is described by a single nonlinear ordinary differential equation obtained by using single mode Galerkin truncation of a set of nonlinear partial differential equations. By assuming that the ratio of the crank radius to the length of the connecting rod is small, the transverse deformation is approximated by an asymptotic series expansion, using the method multiple scales, in terms of this ratio. Several resonances, including the primary resonance, the principal parametric resonance, and various super- and sub- harmonic resonances, are investigated in detail. The analytical results are confirmed by extensive numerical simulations.

The analytical results show that response depends, in a nontrivial manner, on several parame-

ters. These are: the ratio of the crank radius to the length of the connecting rod, the ratio of the slider mass to the connecting rod mass, the ratio of the crank speed to the fundamental frequency of flexural vibration, an internal material damping parameter and slider friction. The effects of these parameters on the flexural vibration of the connecting rod are investigated analytically and numerically. The results indicates that the connecting rod possesses a softening cubic nonlinearity, with the mass ratio as the primary source of nonlinearity. The internal material damping has a favorable effect in reducing the amplitude of the response and the instability regions for several resonances. The slider friction has an adverse effect in that it increases the instability regions for several resonances, but it also has a favorable effect in reducing the amplitude of the response. It is also determined that the response amplitude depends in a non-monotone manner on the mass ratio in certain superharmonic resonance cases. This may be useful in design considerations.

In summary, this approach demonstrates how one can, in some relatively simple cases, obtain useful information about the response of flexible links in mechanisms without resorting to finite element methods or brute-force simulation studies. With analytical expressions in hand, it is a much simpler task to determine the influence of system parameters on the response, especially in the important frequency range where resonances occur.

Detailed results from this study can be found in the references given below.

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CHAOTIC MOTION OF A GYROSTAT SATELLITE IN A CIRCULAR ORBIT

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Abstract

According to Kelvin, a gyrostat consists of two parts: a small axisymmetric rotor inside a large platform. The rotor spins about its axis of symmetry with respect to the platform. For practical applications we find the platform either at rest or rotating very slowly, e.g. in the locked rotation. The rotor spins very fast and serves to stabilize the satellite's attitude.

In recent studies [1], it is found that there exists chaotic motion for spinning satellites consisting of one rigid body only, i.e. the platform, in a circular orbit. In this paper, we extend our studies to the attitude motion of spinning gyrostat satellites in a circular orbit. Here, we only consider the gyrostat satellites with one single axisymmetric rotor inside an axisymmetric platform under the action of the gravitational torque, where the axes of symmetry of platform and rotor coincide. The full nonlinear equations of attitude motion of spinning gyrostat satellites are derived and their Hamiltonian is established, and subsequently numerically investigated. Various dynamic behaviors of spinning gyrostat satellites, e.g. periodic, quasiperiodic, and chaotic are studied via the Poincaré map technique. The effect of rotor speed on the attitude motion is also studied. It is shown that the spin velocity of the rotor has a significant effect on the dynamic behavior of spinning gyrostat satellites, e.g. a chaotic motion will become a regular motion as the spin velocity of the rotor increases.

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SESSION 12
APPLICATIONS TO STRUCTURAL ELEMENTS
WEDNESDAY - 1030 - 1210
JUNE 10, 1992

On the Dynamic Behavior of a Flexible Beam carrying a Moving Mass

F. Khalily * M. F. Golnaraghi † G.R.Heppler ‡

Abstract

The analysis of the dynamical behavior of a flexible structure caused by the motion of masses that are internal to the system has been of interest for many years [1, 2, 3, 4, 5, 6]. The main difficulty encountered in this problem lie in the fact that the system of equations of motion are nonlinear integral/partial differential equations and involve discontinuities.

In this paper a system which consists of a clamped-free beam and a single moving mass is considered. The mass is induced to move by an applied force as opposed to the usual case of a prescribed motion, in which case the position of the moving load is known and is independent of the motion of the structure. An example of this situation is the behavior of a bridge due to the motion of vehicles across it [5,6]. However, the system to be discussed in the proposed paper has the unique characteristic that the motions of the mass and the beam are coupled.

Given the above system, a mathematical model is developed and two coupled integral/partial differential equations which describe the motion of the beam and the mass are obtained. Since the equations of motion are coupled, nonlinear and have both time and space dependencies, it is difficult to solve them analytically. Furthermore, the equations are difficult to solve numerically in their original form. Therefore in our approach the set of partial differential equations were reduced to a set of ordinary differential equations by using the Method of Virtual Work and adopting a trial solution that assumes separation of time and space. Under this assumption the solution for the beam motion is deemed to have the following form:

$$W(x,t) = \sum_j \Phi_j(x) T_j(t) \tag{1}$$

where $W(x,t)$ represents the deflection of a point on the beam which is located at distance x away from the clamped end of the beam at time t, $\Phi_j(x)$ is the j^{th} modal function which satisfies the boundary and transient conditions and $T_j(t)$ is the modal coordinate for the j^{th} mode. In this approach $\Phi_j(x)$ is assumed to be a known function, therefore by replacing W(x,t) by the series (1) the explicit spatial dependency of the equations vanishes and a set of ordinary differential equations which have only explicit time derivatives are obtained. As it was mentioned before, knowing the modal functions is crucial to solving the equations of motion. For the problem to be discussed in this paper the modal function, $\Phi_j(x)$, is not only a function of position x but

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it also depends on the changing position of the mass. Therefore in order to calculate the exact modal function the beam is divided into left and right parts with respect to the mass:

$$\Phi_{jL}(x) = A_1(x_m)\sin(\beta x) + A_2(x_m)\cos(\beta x) + A_3(x_m)\sinh(\beta x) + A_4(x_m)\cosh(\beta x) \quad 0 \leq x \leq x_m \tag{2}$$

$$\Phi_{jR}(x) = B_1(x_m)\sin(\beta x) + B_2(x_m)\cos(\beta x) + B_3(x_m)\sinh(\beta x) + B_4(x_m)\cosh(\beta x) \quad x_m \leq x \leq L \tag{3}$$

The functions $A_i(x_m)$, $B_i(x_m)$, $i = 1, \ldots, 4$(x_m is the position of the mass) are determined using the boundary conditions and the continuity conditions at the mass. The overall displacement field must have a discontinuous second and third derivative. The discontinuous second derivative is due to the inertial moment supplied by the mass which is modelled as a rigid body, while the third derivative condition arises from the change in the internal shear force at the location of the mass due to the inertial forces. The discontinuity conditions in addition to the boundary conditions applied to equations (2) and (3) result in a transcendental equation whose solution can be used to obtain the natural frequencies of the beam. Since all the above functions and also the natural frequency of the system depend on the position of the mass, a computer program has been developed to calculate these parameters as the mass moves. Hence, we can obtain an exact modal function at any instant and the form of that function depends on the position of the mass.

Once the mode shapes have been determined it is possible to determine the dynamical behavior of both the mass and the beam as the mass moves under the applied load. It is then possible to use this simulation to assist in the design of control strategies that will minimize the effect of the motion of the mass or that will utilize the motion of the mass to damp out unwanted vibrations in the beam.

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A GEOMETRICALLY-EXACT BEAM THEORY ACCOUNTING FOR WARPINGS AND 3-D STRESS EFFECTS

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A nonlinear curved and twisted beam can be used to model helicopter rotor blades, aviation propeller blades, turbine blades, aircraft wings, arm-type positioning mechanisms of magnetic disk drives, robot manipulators, helical springs, etc. Moreover, beam-type elements are usually used in material characterization as well as basic structural elements.

A geometrically-exact nonlinear beam model is developed for naturally curved and twisted solid composite rotor blades undergoing large vibrations in three-dimensional space. The theory accounts for in-plane warpings due to bending and extensional loadings, out-of-plane warpings due to shearing and torsional loadings, elastic couplings among warpings, and three-dimensional stress effects by using the results of a two-dimensional, static, sectional, finite-element analysis. Also, the theory fully accounts for extensionality, initial curvatures, and geometric nonlinearities by using local stress and strain measures and an exact coordinate transformation, which result in fully nonlinear curvature and strain-displacement expressions. Six fully nonlinear equations of motion describing one extension, two bending, one torsion, and two shearing vibrations of composite beams are obtained by using a combination of the extended Hamilton principle and the concept of virtual local rotations. The equations display linear elastic couplings due to structural anisotropy and initial curvatures and nonlinear geometric couplings. The theory contains most of beam theories as special cases. Moreover, the formulation is based on an energy approach, but the derivation is fully correlated with the Newtonian approach and the final equations of motion are put in compact matrix form.

Geometric nonlinearities due to large rotations couple the equations of motion. Moreover, because of anisotropy and initial curvatures, bending, torsional, and extensional vibrations of a composite beam can be linearly coupled. In the presence of linear elastic and nonlinear geometric couplings, the response of a beam can be a very complicated three-dimensional motion. The forced nonlinear vibration of a symmetrically laminated graphite-epoxy composite beam is investigated. The analysis focuses on the case of primary resonance of the first flexural-torsional mode when its frequency is approximately one-half the frequency of the first out-of-plane flexural mode. Period-doubling bifurcations and chaotic motions are addressed.

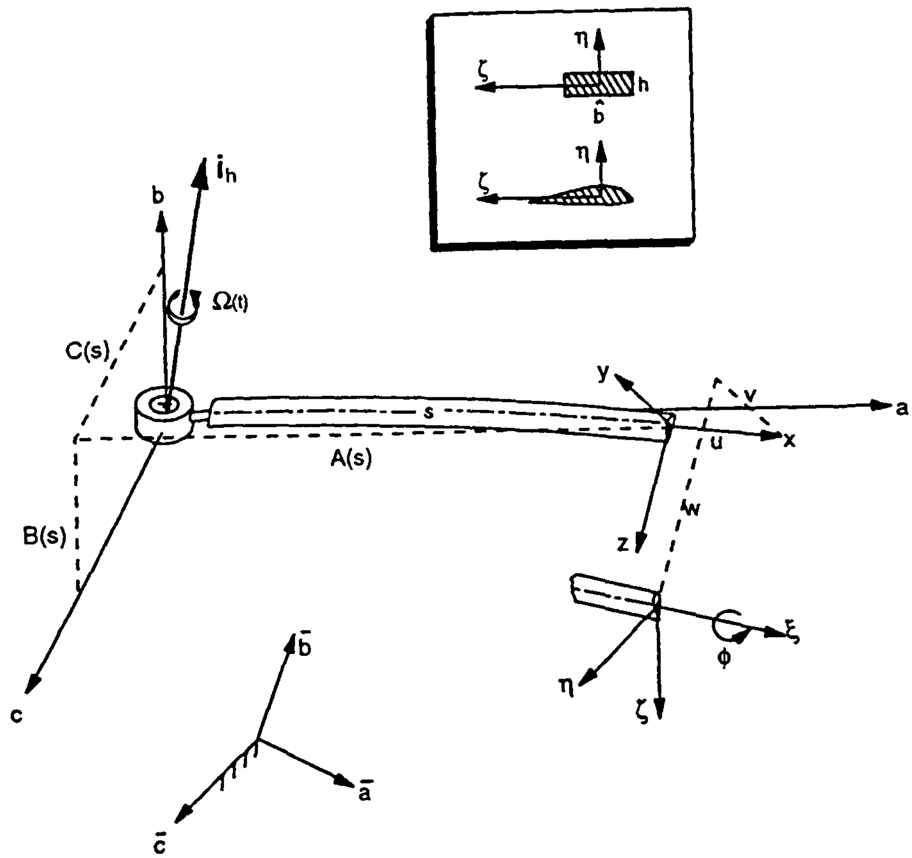

Fig. 1

Experiments on the Nonlinear Resonant Response of Thin Elastic Plates

by S.A. McCabe, P. Davies, S.I. Chang, and A.K. Bajaj,

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The nonlinear vibratory behavior of thin elastic plates has been studied in detail by many researchers [1]. When the natural frequencies of the plate are well separated and the plate is excited by a sinusoid with a frequency close to a resonance frequency, the behavior of the plate can be modeled as an oscillator with a cubic stiffness term. Further theoretical studies have been carried out on thin plates where two or more modes of the plate have coincident frequencies. When such plates are excited harmonically near the coincident natural frequency, the response of the plate can be reasonably well modelled as a nonlinear multi-degree-of-freedom system of order equal to the number of coincident modes. Two of the authors have studied the behavior of this simplified model for the case where the (1,2) and (3,1) modes of a thin pinned-pinned rectangular plate are coincident [2]. Expressions have been derived, through the method of averaging, that show the slow time evolution of the amplitudes of the modal responses of the plate as a function of the excitation frequency and amplitude, and the modal damping.

Having predicted theoretically the behavior of thin rectangular plates, it is important to ascertain if similar behavior can be observed in an actual experiment. Ultimately, the goal is to model and analyze the behavior of nonlinear physical systems. Therefore, it is important to show that the theoretical models are useful for the analysis of plates in real systems. A rig has been constructed to examine the behavior of thin plates under in-plane tension loading. The rig is similar to that constructed by Yasuda and Asano [3]. The behavior of the plate in the experimental rig differs from the theoretical behavior because there are variations in the boundary conditions, the tension is not uniform across the plate, and the actual boundary conditions are closer to those of a clamped-clamped, rather than a pinned-pinned, plate. The plate was placed in tension so that the (1,2) and (3,1) modes had nearly coincident natural frequencies and the frequencies of the (1,1) and (2,1) modes were well separated.

Intuitively, it can be expected that the plate behavior predicted by the theoretical models will be close to that observed when either of the first two modes is excited near its resonance frequency, because the behavior is relatively insensitive to the mode shape. However, for harmonic excitation of coincident modes, the nonuniformities in the plate boundary conditions and tension, which give rise to nonuniformities in the plate mode shapes, can cause severe problems. In the development of the theoretical equations the response of the plate was decomposed into a contribution from each of the two modes: $w(x,y,t) = X_1(t).\ F_{12}(x,y) + X_2(t).F_{31}(x,y)$ where $F_{mn}(x,y)$ denotes the mode shape of the (m,n) mode, and (x,y) are the co-ordinates of the measurement position on the plate. w(x,y,t) can be measured at different locations and, if the mode shape is known, $X_1(t)$ and $X_2(t)$ can be calculated. Ideally, one measurement location would be on a nodal line of the (1,2) mode and the other on a nodal line of the (3,1) mode. It is therefore crucial that $F_{mn}(x,y)$ be known so that $X_1(t)$ and $X_2(t)$ can be observed independently and the theoretical predictions can be directly compared with the measurements. Because of the problems associated with boundary conditions and tension, however, it may only be possible to get qualitative agreement between the analytic predictions and the measured responses.

Ultimately, the goal of the research is to observe the complex motions predicted by the coincident mode theory. As a preliminary step, the behavior of the plate close to its first and second natural frequencies was examined in detail. Mode shapes were calculated from impact test measurements of the plate and also from exciting the system sinusoidally at the resonance frequencies. Nonlinear Autoregressive Moving Average (NARMAX) system identification techniques [4] were employed to confirm the theoretical Duffing's model of the plate in these modes of excitation and also to estimate the physical parameters of the system. This was done at several locations over the plate to investigate variations due to nonuniformities in the test rig construction. The analysis was performed for both the (1,1) and (2,1) modes to investigate the increase of sensitivity with frequency of the mode shapes to nonuniformities in the boundary conditions. Graphs of amplitude of response versus excitation frequency were also plotted from the experimental data and compared to those predicted theoretically. Preliminary analysis of the coincident mode behavior under sinusoidal excitation is also included.

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PARAMETRICALLY EXCITED NONLINEAR VIBRATIONS OF COMPOSITE FLAT PANELS EXHIBITING INITIAL GEOMETRIC IMPERFECTIONS AND INCORPORATING NON–CLASSICAL EFFECTS

By

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ABSTRACT

The non–linear transverse response of flat composite panels subjected to in–plane periodic compressive edge loads is investigated. The influence played by a number of non–classical effects, namely by transverse shear, higher–order effects, unavoidable initial geometric imperfections and the character of in–plane boundary conditions is emphasized and a number of conclusions are outlined.

In contrast to the case of geometrically perfect panels where the parametric response problem is governed by homogeneous differential equations with periodic coefficients and where the transverse vibrations are exhibited over certain regions of the load amplitude–frequency space, in the case of initially imperfect panels the transverse vibrations occur at all load amplitudes and frequencies.

The results obtained in the case of columns with initial curvature excited by periodic axial loads [see Mettler (1967) and Stevens (1969)] reveal that, as in the case of their perfect counterparts, the resonances still appear at $\theta \approx 2\Omega/k$ ($k = 1,2,3 \ldots$). Moreover, the theoretical and experimental results reveal that in the vicinity of odd–order resonances ($k = 1,3,5 \ldots$), the imperfections have little effect on the response whereas in the vicinity of the even–order resonances ($k = 2,4,6 \ldots$),

the amplitude of imperfect columns increases rapidly as the boundary of the corresponding instability region for the perfect column is approached.

Since the second–order resonance ($\theta \approx \Omega$) is the most significant of the even–order resonances in the sense that it yields the largest vibrational amplitudes [Mettler (1967)], this case will be considered in conjunction with the parametric resonance of shear deformable panels exhibiting initial geometric imperfections. In addition, for the sake of completeness some additional results which concern the influence of transverse shear deformability and the character of in–plane boundary conditions on the amplitude–frequency behavior of perfect panels at the first parametric resonance region ($k = 1$) will be presented.

To the best of the authors' knowledge, this paper addresses for the first time the problem of the influence of initial imperfections coupled with that of transverse shear flexibility on the parametrically excited nonlinear vibrations of composite structures.

Large Flexural Vibrations of Thermally Stressed Layered Shallow Shells

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Abstract

Based on a procedure developed for isotropic plates /1/, the scope of this paper is to derive a unifying representation of the influence of large amplitudes on forced vibrations of shallow shells with arbitrarily shaped polygonal planform. Moderately thick shells composed of multiple perfectly bonded layers are considered. A distributed lateral force loading is applied to the structure, and additionally, the influence of a static thermal prestress, corresponding to a spatial distribution of cross-sectional mean temperatures, is taken into account. The shell edges are assumed to be prevented from in-plane motions and are simply supported. The geometrically nonlinear problem of large amplitude vibrations is treated by means of the Karman-Tsien theory, modified by the generalized Berger-approximation that is suitable for shallow shells with immovable edges. Shear deformation is considered according to Mindlin's kinematic hypothesis.

Furthermore, in the special case of laminated shells made of isotropic layers with physical properties symmetrically disposed about the middle surface, a correspondence to moderately thick homogeneous shells is found.

A multi-mode expansion in the Galerkin procedure is applied to the governing differential equation of the nonlinear forced shell vibrations, using the eigenfunctions of the corresponding linear plate problem. A coupled set of ordinary time differential equations for the generalized coordinates with cubic as well as quadratic nonlinearities results.

For a qualitative study, the nonlinear steady-state response of shallow shells subjected to a time-harmonic lateral excitation is investigated. With respect to the phenomena of primary resonance, a single-term approximation is used and the solution of the corresponding problem is obtained by means of the "perturbation method of multiple scales" according to Nayfeh and Mook /2/. A unifying non-dimensional representation of the nonlinear frequency response function is presented, that is independent of the special shell planform. The individual shape of the shallow shell enters the transformation into real time through the linear natural frequencies, or equivalently, through the linear eigenvalues of an effectively prestressed membrane, that is as shaped as the shell's base plane.

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SESSION 13
MULTIBODY DYNAMICS I
WEDNESDAY - 1330 - 1510
JUNE 10, 1992

Experimental High Speed Response of a Slider Crank

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A slider crank mechanism has been constructed for the expressed purpose of investigating very high speed elastic phenomena. These features include 1) a combination flywheel and crank that possesses a large moment of inertia and adjustable crank length, 2) a 1/16 inch thick rod made of high strength Aluminum alloy to allow for deflections that will exceed 10 percent of the rod length without yielding, and 3) a piston connected by nearly frictionless bearings to steel rods that serve as the piston slide axis. The mechanism is driven by a 1/2 HP DC motor whose speed is controllable by a potentiometer. A steel case with a plexiglass shield for viewing enshrouds all moving parts for protection of the operators. Signals from two or three strain gages mounted onto the rod (fed through a strain gage conditioner), and magnetic signals that sensed crank rotations, were converted from analog to digital and stored on a 286 computer.

A series of experiments were performed, beginning with a very small crank length (approximately 1/40 the rod length), and increasing to a crank length of approximately 15 percent of the rod length. In all cases, response at any given speed increased with crank length. For the very small crank, speeds approaching 2000 rpm (80% of the rod natural frequency) were obtained. In all experiments, a zone of increased response occurred at crank speeds near 1/2 the natural frequency of the rod. For intermediate size cranks, period doubling occurred at speeds beyond the first natural frequency. In one instance, it was possible to slowly increase the speed and pass through the zone of period doubling, followed by a zone of vibrations again equal to the crank period. The onset of period doubling occurred at lower crank speeds when the crank length increased. At the largest crank sizes period doubling disappeared. In those cases the rod yielded at crank speeds below those necessary to induce period doubling.

The operating mechanism will be videotaped using a high speed camera for presentation at the conference.

Steady State Response of a Slider Crank with Flexible Rod

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The flexible rod slider crank mechanism has been the subject of numerous investigations of response and stability. The mechanism includes a rigid crank that rotates at a constant angular velocity, a flexible rod modeled as an Euler-Bernoulli beam, a piston, and an external force on the piston. In this work an investigation of nonlinear steady state stability and bifurcations of this system is performed based upon a one mode approximation to beam bending. Based upon response investigations presented by other authors, which revealed that flexible rod transverse vibration was the same period or twice the period of the crank rotations, a steady state solution of the following form was assumed:

$$a_0 + a_{1/2}\cos\frac{t}{2} + b_{1/2}\sin\frac{t}{2} + a_1 cost + b_1 sint + \ldots + a_5 cos5t + b_5 sin5t$$

Harmonic balance is next applied to find steady state solutions necessary for linearization. Nonlinear and linear terms in the ODE that would otherwise burden harmonic balance are incorporated into another truncated series like Equation (1). The coefficients of both equations are solved using the computational Harmonic Balance/Fast Fourier Transform (HBM/FFT) scheme, which includes a Newton-Raphson solver for solution of nonlinear algebraic equations in the trigonometric coefficients of Equation (1). Linearization is performed about this steady state solution. The perturbational equation contains periodic coefficients, and so monodromy matrix methods were applied to computationally determine the eigenvalues that determine stability. Plots of crank speed versus vibration amplitude were presented, that also pinpoint saddle-node (jump) bifurcations, flip (period doubling) bifurcations, and regions of amplified response.

Six cases were studied. In the first case, the crank is small, with no piston mass or external gas force. The response was amplified near crank speeds equal to the first natural frequency of the rod in bending. In the second case, the piston mass is included. The peak response occurred at a speed less than the first natural frequency, reflecting a speed induced decreased natural frequency. In Cases 3 through 5, the crank size was increased from small (as in perturbational methods) to 1/2 the rod length, and the external gas force added to the piston in all three cases. The larger crank size induced more severe nonlinearities at lower crank speed. Saddle-node and period doubling bifurcations occurred in Cases 4 and 5. In Case 5, noticeable "irregular resonance" due to amplified superharmonics occurred at crank speeds near 1/3 and 1/4 the rod natural frequency.

The Inter-relation between Multidbody Dynamics Computation and Nonlinear Vibration Theory

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Abstract

This paper revisits the formulation and numerical solution of the Autoparametric Vibration Absorber (AVA) introduced by Haxton and Barr (1971). The study is mainly applied with reference to multibody dynamics theory. Using Lagrange multipliers the equations of motion are written in multbody dynamics terms giving a coupled set of differential and algebraic equations of motion to be solved by a Gear stiff integrator and the ADAMS program. The paper will show the equivalence between the equations of motion for two formulations; the Haxton and Barr formulation using minimal coordinates and the ADAMS formulation using Lagrange multipliers and constraint equations. It will explain the physical interpretation of various nonlinear forces found in the Haxton and Barr equations, showing their evolution from a constraint-equation. The model will evaluate data over a sequence of damping ratios and applied forcing frequencies in agreement with the Haxton and Barr test parameters. Finally, response plots will show how the Haxton and Barr numerical approximation can be derived by ignoring one nonlinear force term in their formulation.

The analysis introduces the concept of *constrained flexibility*. This is a modeling technique that combines the lateral elastic stiffness of a cantilever beam with an algebraic nonlinear axial displacement constraint between two rigid bodies. This technique follows from the constraint application required to reduce the system to the necessary two degrees of freedom and suggests future utility in the modal analysis of multibody systems.

The analysis also includes the use of an *instantaneous energy checking function* to improve integration parameter selection in the numerical scheme. Finite difference methods for solving continuous, ordinary differential equations may introduce *spurious solutions* . This complex source of trouble can be a function of stepsize, error tolerance and predictor polynomial order combined with nonlinear equations. Consequently, the equations of motion and constraints are not sufficient to insure computed accurate answers. The energy balance checking function uses computed body velocities and positions from the numerical solution. It also monitors the accumulated or global energy error at any simulation time. If the energy error is large artificial energy is introduced by the numerical method and adjustment is made to the integration control parameters.

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Acknowledgement

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STEADY-STATE ANALYSIS OF LARGE SCALE MULTIBODY SYSTEMS WITH SPECIAL REFERENCE TO VEHICLE DYNAMICS

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ABSTRACT

In some applications of multibody systems, it is necessary to determine the steady state response of a system under given conditions. For highly simplified problems, it may be possible to write and solve the steady state equations in explicit form. However, for large scale problems without too many simplifications, finding a closed form solution is not possible, and furthermore, determining a numerical solution may not be easy if the problem is not properly formulated. This paper presents a systematic method for formulating and numerically solving the steady state equations for multibody systems. The equations of motion are first written in terms of relative joint coordinates using velocity transformation formulas. Then, conditions for a given steady state response are introduced into the equations of motion in order to obtain the steady state equations. For example, for a nonperiodic steady state response, accelerations can be generally zero, velocities can be zero or constant, and in addition, specific relationships can exist between some or all of the coordinates. Such conditions transform the differential equations of motion into a set of nonlinear algebraic equations where the unknowns are all of the coordinates and some of the velocities. These equations are solved numerically using the Newton-Raphson method. If the original equations of motion are expressed in terms of a large set of dependent Cartesian or absolute

coordinates, the steady state conditions can not be stated easily and, hence, the procedure for finding the steady state configuration can become cumbersome. However, when the equations are expressed in terms of a relative set of joint coordinates, these relationships are rather obvious and simple to introduce.

One applications of this methodology is in vehicle dynamics. A vehicle maneuvering a circular path with a constant speed is one example of the steady state response. In this paper, a complete multibody model of a four-wheel vehicle including the suspension and the steering systems is described. The interacting nonlinear forces and moments between the tires and the road, due to the longitudinal slip, lateral slip, and camber angle are incorporated into the model. The steady state response of the vehicle for different speeds and different steering angles are presented. Also the two-wheel and four-wheel steering responses are compared. The results are further compared for accuracy and efficiency against those obtained from solving the dynamic equations of motion.

SESSION 14

COUPLED OSCILLATORS II

WEDNESDAY - 1530 - 1710

JUNE 10, 1992

ITERATED MAPS IN THE PERIODIC RESPONSE OF A TWO DOF ELASTOPLASTIC SYSTEM

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Quite a number of papers have been written on the periodic solutions for multi-dof nonlinear elastic systems, usually of a low order, using both perturbation and harmonic balancing techniques. The response appears richer than for the one dof oscillator. Moreover, besides the super and subharmonic resonances and the occurrence of chaotic motion, phenomena due to internal resonance and combination resonance are also possible.

On the contrary only few papers exist regarding hysteretic oscillators with more than one dof. Here the periodic solutions are only found with the harmonic balance method, that in its standard formulation - one frequency only - does not allow to catch the whole aspect of the response. In particular the internal resonance effects are not evidenced, and then this standard formulation can not be used when there is an important resonant coupling. This inconvenience can be overcome if more frequencies are taken into account. That, however, brings to complex algebraic equations whose solution has to be determined numerically and then a qualitative analysis becomes more difficult.

In this work a two degree elastoplastic system under harmonic excitation is studied with a numerical technique which allows a qualitative analysis as well. The assumed tool is the Poincaré map **P** together with some other return maps connected with it. In such a way the periodic response is studied, at least formally, with purely algebraic procedures. The Poincaré map for a periodically forced system is simply defined as the function which maps the phase space in itself giving the position that a point p reaches after the integration of the motion equation over one period of the external force time. To eliminate as more sources of errors as possible, the numerical integration is carried out exactly. That is possible because in each plastic phase - i.e. in each combination of yielded plastic forces - the elastoplastic system behaves linearly. The only difficulty lies in the identification of the transition to the various plastic phases that requires solution of simple nonlinear equations. The periodic solutions are the fixed points of **P** and the stability is simply checked looking at the eigenvalues of the tangent map **TP** of **P**. The phase space has dimension four, and is defined by the velocity v_i of the two masses and by the two forces f_i of the elastoplastic elements. The displacement, being unessential for the constitutive relationship which is expressed in incremental form as $\dot{f}_i = h(f_i, sign(v_i))v_i$ (i=1,2), is not assumed as an independent variable. Its history can simply be obtained by integration of the velocity history.

Some problems rise in the use of the Poincaré map $\mathbf{P}$ as a matter of fact that it is not a diffeomorfism. More precisely $\mathbf{P}$ is continuous with its derivative, with the exception of a limited number of points, but it is not invertible. That because the condition of Cauchy-Lipschitz for a unique solution to exist, is violated. Another difficulty is due to the existence of boundaries in the phase space as the f_i cannot exceed their limit values. These problems are by-passed with the introduction of alternative return maps that have a lower dimension space as domain. Where the periodic motion under examination requires to pass the plastic value for only one force f_i then the phase space dimension is reduced by one otherwise it is reduced by two. The state variable that is eliminated is precisely the force that reaches the limit plastic value.

The examination of the frequency response curves for the two dof system here considered, obtained varying the frequency ω of the external force, shows as the influence of internal resonances can not be disregarded unless the modal coupling is very low. Differently from the elastic nonlinear case the effect of internal resonances is shown only in a superharmonic fashion. Namely, if the ratio of the two natural frequencies ω_1 and ω_2 is close to three and the external force is resonant with the first mode, $\omega \simeq \omega_1$, a components 3ω comes up. Instead if the resonance for the second mode is assumed, $\omega \simeq \omega_2$, the component $1/3\omega$ is not shown whichever the initial conditions are. Another peculiarity of the hysteretic systems is the way the energy is distributed between the two modes. In presence of internal resonancese, for $\omega \simeq \omega_1$, there is a sharp energy absorption by the 3ω component at the expense of the fundamental one. That results in a sharp decrease of the amplitude of the first mode. Consequently the frequency response curves show two apparent maxima both in the fundamental mode and in the total response. This situation gives problems in system identification applications where the frequency response curves are obtained experimentally because the two peeks are seen as two different natural frequencies.

The stability analysis, made by checking the eigenvalues of $\mathbf{TP}$ indicates that the frequency response curves are always stable. Then, besides of being single valued, they do not present bifurcations corresponding to asymmetric or quasiperiodic solutions. The possibility for the existence of insoles cannot be excluded at this phase of the research, as the occurrence of subharmonic orbits cannot be excluded as well. However an extended numerical survey has not shown something like that yet.

Analytical Construction of the Two-Parameter Family of Quasiperiodic Solutions in the Autonomous System.

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Consider the following nonlinear dynamical system

$$\ddot{x}_s+\lambda_s^2(\epsilon,\mu)x_s = F_s(x_1,\dots,x_n,\epsilon,\mu),$$
$$s = 1,\dots,n, \tag{1}$$

For $\epsilon=0$ *and* $\mu=0$ *we have* $\lambda_s=\omega_s$, *and the frequencies* ω_s *are incommensurable and positive, i.e.*

$$((k,\omega)) = k_1\omega_1+k_2\omega_2+\dots+k_n\omega_n \neq 0; \tag{2}$$

ϵ *and* μ *are certain small and independent perturbation parameters with* $\epsilon\in[0,\epsilon_0]$ *and* $\mu\in[0,\mu_0]$. *We want to find the two parameter family of quasiperiodic orbits defined as follows*

$$x_s = Q^s(\phi_1,\dots,\phi_n,\epsilon,\mu),$$
$$\dot{\phi}_s = \omega_s,$$
$$Q^s(\phi_1,\dots,\phi_n,\epsilon,\mu)$$
$$= Q^s_{00}+\epsilon Q^s_{10}(\phi_1,\dots,\phi_n)$$
$$+\mu Q^s_{01}(\phi_1,\dots,\phi_n)+\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\epsilon^k\mu^l Q^s_{kl}(\phi_1,\dots,\phi_n),$$
$$\lambda_s = \omega_s+\epsilon\eta_{10}+\mu\eta_{01}+\sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\epsilon^k\mu^l\eta^s_{kl},$$
$$s = 1,\dots,n, \tag{3}$$

where:

$$Q^s_{00} = B^{00}_s\cos\phi_s+C^{00}_s\sin\phi_s,$$
$$B^{00}_s = B^1_s+B^2_s,\ C^{00}_s = C^1_s+C^2_s,$$
$$Q^s(\phi_1,\dots,\phi_n,\epsilon,\mu) = Q^s(\phi_1+2\pi,\dots,\phi_n+2\pi,\epsilon,\mu). \tag{4}$$

We take the arbitrary condition

$$\frac{\partial Q^s(0,\dots,0,\epsilon,\mu)}{\partial\phi_s} = 0,\ s = 1,\dots,n, \tag{5}$$

and from a first reccurent equation we find

$$P^s_{10}(B^1_1,\dots,B^1_n) = \int_0^{2\pi}\dots\int_0^{2\pi} F^s_{10}\sin\phi_s d\phi_1\dots d\phi_n = 0, \tag{6}$$

$$\eta^s_{10} = \frac{1}{(2\pi)^n\omega_s B^1_s}\int_0^{2\pi}\dots\int_0^{2\pi} F^s_{10}\cos\phi_s d\phi_1\dots d\phi_n = 0, \quad s=1,\dots,n, \tag{7}$$

and

$$P^s_{01}(B^2_1,\dots,B^2_n) = \int_0^{2\pi}\dots\int_0^{2\pi} F^s_{01}\sin\phi_s d\phi_1\dots d\phi_n = 0, \tag{8}$$

$$\eta_{01} = \frac{1}{(2\pi)^n\omega_s B^2_s}\int_0^{2\pi}\dots\int_0^{2\pi} F^s_{01}\cos\phi_s d\phi_1\dots d\phi_n = 0, s=1,\dots,n. \tag{9}$$

From (6) and (8) we find $B^1_1,\dots,B_n{}^1$ *and* $B^2_1,\dots,B^2_n$, *respectively.* η^s_{10} *and* η^s_{01} *are obtained from (7) and (9).*

The next recurrent equations allows for finding $C_s^{10(01)}$, *whereas* $B_s^{10(01)}$ *are obtained from (5). To achieve a complete ordering of all of the recurrent equations we take the following additional condition*

$$\epsilon^{j-1} < \mu^j, \tag{10}$$

where j is a positive integer. The triangle below gives the ordering from the smallest to the largest asymptotically on each horizontal row, i.e. defines the sequence of recurrent equations

$$\begin{array}{c} \mu \quad \epsilon \\ \mu^2 \quad \epsilon\mu \quad \epsilon^2 \\ \mu^3 \quad \mu\epsilon^2 \quad \epsilon^2\mu \quad \epsilon^3 \\ \mu^4 \quad \epsilon\mu^3 \quad \epsilon^2\mu^2 \quad \epsilon^3\mu \quad \epsilon^4 \end{array} \tag{11}$$

The presented approach is illustrated by the example of two degrees of freedom mechanical system, where the dampers parameters play the role of independent perturbation parameters. Additionally, the catastrophes of tori are classified and discussed.

CONSTRUCTING INVARIANT TORI FOR TWO WEAKLY COUPLED VAN DER POL OSCILLATORS

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A perturbation algorithm is developed for the direct construction of an invariant torus of a weakly coupled autonomous oscillator. The algorithm incorporates an averaging procedure to select the coefficients for the expansion. It is applied to a system of nonlinearly coupled van der Pol equations.

$$\ddot{z}_1 + \mu_1^2 z_1 = \epsilon(1-z_1^2-az_2^2)\dot{z}_1,\ \ddot{z}_2 + \mu_2^2 z_2 = \epsilon(1-\alpha z_1^2-z_2^2)\dot{z}_2 \tag{1}$$

where $a > 0$, $\alpha > 0$ and $\mu_1 > 0$, and $\mu_2 > 0$ are linearly independent over the integers. Introduce coordinates r and ϕ into (3). The new system becomes

$$\dot{\phi} = d + \epsilon\Phi(\phi,r), \quad \dot{r} = \epsilon R(\phi,r). \tag{2}$$

Define the average of $R(\phi,r)$ from (5) by $R_0(r)$. Φ and R in (2) are periodic in ϕ with vector period $(2\pi/\mu_1,2\pi/\mu_2)^T$ in $(\phi_1,\phi_2)^T$. Let $r^{(0)}$ be a vector such that $R_0(r^{(0)}) = 0$ and the real parts of the eigenvectors of $\partial R_0/\partial r$ have nonzero real parts. Then there is a vector function $r(\phi,\epsilon)$, periodic in ϕ with vector period $(2\pi/\mu_1,2\pi/\mu_2)^T$, $r(\phi,0) = r^{(0)}$ and $r(\phi,\epsilon)$ is an invariant torus for (2).

To begin with the construction assume an expansion for $r = r(\phi,\epsilon)$ in the form

$$r = \sum_{n=0}^{\infty} \epsilon^n r^{(n)}(\phi) \tag{3}$$

where we seek $r^{(n)}(\phi)$, $n = 0, 1, \ldots$, each with vector period $(2\pi/\mu_1, 2\pi/\mu_2)$. Substituting this in (2) gives the following sequence of partial differential equations

$$\frac{\partial r^{(0)}}{\partial \phi} d = 0, \qquad \frac{\partial r^{(1)}}{\partial \phi} d = R(\phi, r^{(0)}) - \frac{\partial r^{(0)}}{\partial \phi} \Phi(\phi, r^{(0)}), \tag{4}$$

$$\frac{\partial r^{(i)}}{\partial \phi} d = \sum_{k=1}^{i-1} \sum_{n_1+\ldots+n_k=i-1} \frac{1}{k!} \frac{\partial^k R}{\partial r^k} (\phi, r^{(0)}) r^{(n_1)} \ldots r^{(n_k)}$$

$$- \sum_{n=0}^{i-2} \frac{\partial r^{(n)}}{\partial \phi} \sum_{k=1}^{i-n-1} \sum_{n_1+\ldots+n_k=i-n-1} \frac{1}{k!} \frac{\partial^k \Phi}{\partial r^k} (\phi, r^{(0)}) r^{(n_1)} \ldots r^{(n_k)}$$

for $i \geq 2$. Now define the matrix

$$\int_0^{2\pi/\mu_1} \int_0^{2\pi/\mu_2} \frac{\partial R}{\partial r} (\phi, r^{(0)}) \, d\phi_1 \, d\phi_2 \tag{5}$$

Theorem. If $R_0(r^{(0)}) = 0$ and (5) is nonsingular then (4) can be solved sequentially for $r^{(0)}, r^{(1)}, \ldots$, each of vector period $(2\pi/\mu_1, 2\pi/\mu_2)$.

A numerical study is done of some of the characteristics of the flows on the torus. These include a comparative analysis of integrating the van der Pol system on the torus with integrating the phase equations on the approximate torus. A Fourier analysis of the dominant frequencies in the approximate solution as well as a computation of the rotation number of the phase equations on the torus are performed.

NORMAL MODES FOR WEAKLY NONLINEAR DYNAMICAL SYSTEMS

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Abstract

The concept of normal modes of motion is well developed for linear systems, due to the special features of the linear differential equations which govern their dynamics. These special features allow for a definition of normal modes in terms of eigenvectors (or eigenfunctions) and the expression of an arbitrary system response as a superposition of modal responses (see, for example Meirovitch, 1967). A normal mode motion for a system is one which is governed by a single degree of freedom oscillator.

It is obvious that no complete analogy of linear modal analysis can exist for nonlinear systems, simply because superposition does not hold. However, many of the relevant ideas can be generalized. For example, much work has been done on the existence and stability of normal modes of motion for two degree of freedom, conservative systems (see, for example, Rosenberg, 1966, Rand, 1971, and Vakakis, 1990). The purpose of the present work is to generalize these definitions to a very wide class of systems which includes nonconservative, gyroscopic, and infinite dimensional systems (Shaw and Pierre, 1991a, 1991b). In particular, we (1) formulate a definition of normal modes for a general class of nonlinear systems, (2) develop a constructive technique for obtaining these modes for weakly nonlinear systems and, (3) generate the differential equations which govern the dynamics of the system when it is undergoing a normal mode motion. This development clearly demonstrates the origins of the usual normal modes which exist in linearized systems and it also points out the limitations of modal analysis techniques for nonlinear systems.

In order to extend modal analysis ideas to nonlinear systems, an approach which is fundamentally different from the usual separation of variables and resulting eigensolutions must be adopted. Such an approach is offered by defining normal modes in terms of motions which occur on low (typically two) dimensional invariant manifolds of the system's phase space. Such a motion must be inherently like that of a lower dimensional system, and this is exactly what is desired for a normal mode motion. A constructive technique for generating such manifolds in terms of asymptotic series, without having to solve the equations of motion, is provided by a simple generalization of the method used in constructing approximate center manifolds in bifurcation theory (see Carr, 1981). Using this approach we are able to determine the manifolds which represent normal modes for weakly nonlinear systems. The equations of motion restricted to these manifolds then provide the dynamics of the associated normal modes.

For vibratory systems these manifolds are two dimensional and the modal dynamics on them are governed by second-order, nonlinear oscillators. From these manifolds one can deduce the physical behavior of the system undergoing a purely modal motion and, in particular, the amplitude-dependent mode shapes can be obtained. From the nonlinear modal oscillators one can obtain information about the amplitude-dependent frequencies and decay rates. For an N degree of freedom system (N finite or infinite) there exist N such nonlinear normal mode manifolds. The tangent planes to these manifolds at the equilibrium point are the planes on which the usual modal dynamics of the linearized system take place, *i.e.*, they are the eigenspaces.

The above issues are examined in detail for example problems including the vibrations of systems with finite and infinite degrees of freedom.

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Mode Localization In A System of Two Coupled Beams with Geometric Nonlinearities

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Localized nonlinear normal modes have been shown to exist in discrete nonlinear periodic oscillators [1,2]. In this work, nonlinear mode localization is studied for a system of two identical, linearly coupled, weakly nonlinear, inextensible cantilevered beams. Nonlinear curvature and inertial terms are included in the analysis and the free response of the system is investigated by the method of multiple scales.

Using a Galerkin approximation, the first three linearized modeshapes and natural frequencies of the cantilevered beams are computed. The first phase of the work investigates the free response of the 1st mode approximation. Localized solutions are found which branch at a critical value of the coupling parameter off the antisymmetric mode of the system through an inverse pitchfork bifurcation. For coupling parameters above this critical value, the antisymmetric mode is found to be stable. At the point of bifurcation, the antisymmetric mode loses stability and the two bifurcated localized solutions are orbitally stable.

The second phase of the work investigates the effect of internal resonance (which arises due to a 3:1 ratio between the 3rd and 2nd linearized natural frequencies) on nonlinear mode localization. Various combinations of modes are found in which the oscillations of the 2nd and 3rd modes are mainly confined to only one of the two beams. Thus, the energy of the system is found to be spatially confined during these motions. In addition, other solutions were found in which "weak" localization takes place: that is, only one of the two modes localizes. The stability of these modes is then studied.

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SESSION 15

MULTIBODY DYNAMICS II

THURSDAY - 0830 - 1010

JUNE 11, 1992

MULTIBODY DYNAMICS OF AIRCRAFT OCCUPANTS SEATED BEHIND INTERIOR WALLS

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ABSTRACT

Airworthiness is defined as the ability of an aircraft structure to provide maximum opportunity for survival of its occupant during crash (impact). The complex subject of aircraft design for impact protection is truly interdisciplinary, encompassing a variety of field including biomechanics, multi-body dynamics, occupant motion simulation, impact-related injury criteria and finite element methods of structure analysis, etc. One important aspect of these general studies is to reduce head injuries to an aircraft passenger in case of a head contact with its surroundings. In view of the importance of this, studies of post-crash dynamic behavior of victims and the compliance characteristics of the aircraft bulkhead are necessary in order to determine the mechanisms that cause the head injuries and to keep the Head-Injury-Criteria (HIC) below a level of 1000 [1].

Program SOM-LA/TA (Seat Occupant - Light/Transport Aircraft) developed under the Federal Aviation Administration (FAA) sponsorship [2] incorporates a dynamic model of the human body with a finite element model of the seat structure. It was used as an analytical tool in this paper for determination of the occupant response and the compliance characteristics of the bulkhead in various crash environment. Correlated studies of analytical simulations with sled test results were accomplished . The tests, performed at the Civil Aeromedical Institute (CAMI) [3], corresponded to an occupant head striking the aircraft bulkhead or a panel behind which the occupant was seated. The tests were performed with a part 572 HYBRID II anthropomorphic dummy. Modifications in SOM-LA/TA were

achieved to include an envelope for the occupant and the seat. It was observed that SOM-LA/TA reasonably predicts the HIC for the triangular-shaped pulses, which is the pulse required by the FAA. The cases needed to be further analyzed to obtain the dependency of the results on displacement requirement and compliance characteristics of bulkheads.

A contact force model of the form:

$$F = A(e^{B\delta} - 1) + C\dot{\delta}$$

was used, for which A and B were stiffness coefficients and C was the damping coefficient. For different materials, based on both static and dynamic tests, these coefficients are evaluated from experimental correlations. This model was used for the occupant head contact with any envelope. The coefficients were varied in order to observe how the change in these coefficients affect the HIC and maximum deformation of the front panel. A particular choice of the coefficients A, B, and C were obtained which matched the experimental results from CAMI [3]. A parametric study of the coefficients in the contact force model was then performed in order to obtain a correlation between HIC and coefficients in the contact force model. A measure of optimal values for the bulkhead compliances and displacement requirements was thus achieved in order to keep the possibility of head injury as little as possible. This information could in turn be used in the selection of suitable materials for the bulkhead or interior walls of a passenger aircraft.

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MODELLING OF VEHICLE CRASH TESTS BY A MULTIBODY SYSTEM

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Our knowledge of vehicle behaviour and road safety systems during a crash-test is based on experimental studies. A survey was involved on the modelling of a vehicle and more precisely the front-compartment : the model should allow us to enlarge the conclusions drawn from tests extending the results to different situations.

We designed a multibody model with adapted links simulating the behaviour of a vehicle in different frontal crash configurations. Each body or link is formed of all the elements that during the crash follow an identical deceleration law, that is to say all the elements that are "rigidly linked together". They may represent several components of the real structure. One of our first concerns was to make its computer-aid processing supple and rapid both at the level of creation and execution considering the great variability of the input data.The input informations necessary for dealing with the problem are the geometric and kinematic data of the bodies and the links and the data concerning the mechanical behaviour of the links.

The knowledge of the vehicle geometry, the initial kinematic conditions and the type of crash-test allows us to make the decomposition into bodies and links and thus to generate the corresponding multibody model (figure 1). Each body is considered as a rigid solid characterized by its mass, its inertia tensor, its position and orientation and its kinematics. We have used two types of links : geometric links allowing a relative movement of a body compared with the other one and defined by their application point and their mechanical types, and deformable links defined in their turn by the position data of their application points between the different bodies of the model and their behaviour laws.

Given the complexity of a simulation model, a tool was created taking into account the specific problems of design of this type of application. In fact, we define, starting from appropriate data acquisition masks, the geometric and kinematic characteristics of the bodies and the geometric and mechanical characteristics of the links. This allows us to create in conviviality the adequate data files for analysis of the simulation.

Usually, the methods used for the formation of equations governing the movement of a complex system are based either on the use of general theorems or on the use of techniques of analytical mechanics (Lagrange's and Hamilton's equations). Our idea is to start from a method allowing us to draft these equations in a more synthetical form in

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order to leave most developments at the level of data processing. The motion of a body in space is described by the group D of the displacements in affine space $\mathcal{E}$. Because of its differentiable properties, the group of displacements is a LIE group. Then we use these properties for calculations. The final advantage of this type of method is that it allows a totally intrinsic analytical calculation in the vectorial space of the torques.

Our objective is then to obtain for a given system the analytical equations governing motion as a function of generalized coordinates and its movement for different initial conditions. To this effect, we must put into place the whole explicit mathematical formulation in order to design and achieve the directly programmable algorithms. A supplementary tool was created to recover the file of equations on the motion established by the software for generating equations and to allow the real calculation of the solutions given by the formerly obtained equations.

The tools of simulation were created in the framework of the study on the collision of a vehicle with an obstacle. However, although they take into account the specificities of this case, they are sufficiently supple for allowing the interest in any multibody system study, in particular the simulation of the occupant dynamics.

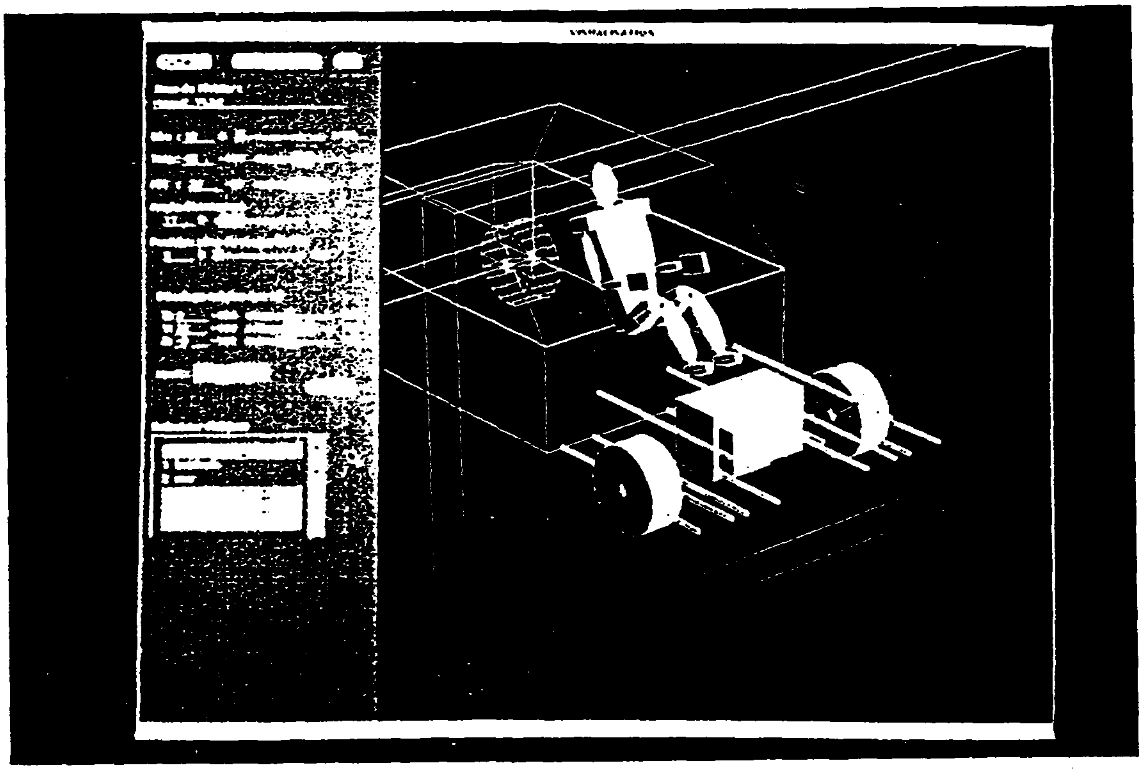

Figure 1

INTERMITTENT MOTION ANALYSIS IN MULTIBODY DYNAMICS USING JOINT COORDINATES AND CANONICAL EQUATIONS OF MOTION

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ABSTRACT

For mechanical systems that undergo intermittent motion, the usual formulation of the equations of motion over the periods of discontinuity is not valid, and a procedure for balancing the momenta is often performed. In this paper, a general canonical form of the equations of motion for multibody systems, in terms of the system relative joint coordinates, is used as the differential equations of motion. These equations are valid for both open- and closed-loop systems. A set of momentum balance-impulse equations are derived in terms of the system's total momenta by explicitly integrating the canonical equations. With known values of the joint coordinates and the momenta right before impact, and also for a given coefficient of restitution, the momentum balance-impulse equations are solved for the change in momenta. The total momenta and the velocities immediately after impact are updated and then the integration of the equations of motion is resumed.

For a central-impact case, the change of velocities is considered in the normal direction to the contact surface. However, for an oblique impact, the change of velocities of the points of contact must be considered both in the normal and tangential directions. Furthermore, the change in relative velocities in a tangential direction would require the inclusion of a friction force between the contacting

bodies. If adequate relationships between the tangential components of relative velocities both before and after impact are not established, an increase in the kinetic energy of the system may be observed due to the assumptions used in predicting the post-collision motion. This phenomenon, which has been pointed out and discussed by other investigators, is thoroughly addressed in this paper.

This paper also presents several examples of open- and closed-loop multibody systems. Impact analysis for these systems during collision is performed and the results are compared against those obtained from previously developed techniques. Due to the small number of momentum balance-impulse equations, the numerical solution of these equations can be obtained efficiently.

On the Dynamics of Tethered Satellite Systems

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Cable-connected satellites are interesting flexible multibody systems in the actual research of orbital space craft. The most important question dealing with the dynamics of such systems is the formulation of the governing equations of motion. Since the choice of the coordinate system effects the complexity of the equation set considerably, and this complexity on the other hand influences the handling within a numerical simulation procedure, the search for the simplest structure of the equations of motion is of decisive importance.

Here, in contrast to the usual propositions in the available literature in this field where inertial geocentric frames and locally attached reference systems are used (see /1/, for instance), a special floating reference frame (called Bucken s frame /2/ and putted to the instantaneous center of mass of the flexible multibody system) is applied and compared with the mentioned other ones. The satellite system is composed of two point-shaped rigid bodies (modeling the shuttle and the subsatellite) connected by a distributed parameter (torsional and bending rigid) tether with finite axial stiffness. The system undergoes an orbital overall motion, additional rigid body pendulum oscillations and longitudinal tether vibrations. As well the generation of the governing nonlinear boundary value problem of the hybrid structure as the reduction to a system of ordinary differential equations by Galerkin s method are discussed in detail. A generalization to e more complicated system with finite bending and torsional rigidities of the tether is also dealt with. Finally, different stationary modes of motion and the corresponding linearized stability equations are briefly addressed. The numerical simulation is not subject of consideration.

All obtained equation sets verify, that also the coordinate system proposed here has several advantages and may be an appropriate starting point studying dynamic problems of tethered flexible satellites (with two endbodies). Particularly, in comparison with the structure of the equations of motion based on a locally attached reference frame, some simplificiations in the final expressions can be stated.

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NONLINEAR MOTION OF AN ARBITRARILY SHAPED SATELLITE IN AN ELLIPTIC ORBIT INCLUDING THE EFFECTS OF DAMPING

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Abstract

For an arbitrarily shaped satellite in an orbit about a poin master, the equation of motion is studied in [1]. Including damping to this system according to power law, one has

$$\frac{d^2\psi}{dt^2} + \frac{d^2\theta}{dt^2} + 3K\sin\psi\cos\psi = -\gamma\left(\frac{d\psi}{dt}\right)^n \tag{1}$$

where θ and ψ are the position angle and libration angle respectively. K is the inertia moment ratio and γ is the coefficient of atmospheric damping. Using Kepler's equation (1) may be written as

$$(1+e\cos\theta)\psi'' - 2e\sin\theta(\psi'+1) + 3K = -\gamma(1+e\cos\theta)^{2n-3}(\psi')^n \tag{2}$$

where ( )' is a derivative with respect to θ.

The time evolution of equation (2) is being studied. For a fixed damping γ and inertia moment ratio K the behaviour of motion changes as the orbit eccentricity e is varied. As an appropriate set of parameters we choose $K = 0.5$ and $\gamma = 0.05$.

For small values of e the motion is periodic encircling the equilibrium point $\psi = 0$, $\psi' = 0$. With increasing e the periodic motion undergoes period doubling bifurcation at $e = 0.132$.

In this paper, the planar motion of a satellite in an elliptic orbit is studied. Occurrence of period doubling as the orbit eccentricity increases is observed.

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SESSION 16
FLOW-INDUCED VIBRATIONS AND
COMPUTATIONAL METHODS
THURSDAY - 1030 - 1210
JUNE 11, 1992

NONLINEAR DYNAMICS OF ARTICULATED CYLINDERS SUBJECT TO CONFINED AXIAL FLOW

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This study is concerned with an articulated system of rigid cylinders interconnected by pins and rotational springs. The cylinders are centrally located in a cylindrical duct and subjected to an external axial flow. The upstream end of the system is pinned while the downstream end is free. Although the formulation is generalized to any number of degrees of freedom (articulations), this study is restricted to models with two and three degrees of freedom. Furthermore, only planar motion is considered.

The equations of motion are derived by the Lagrangian method. The fluid forces on each cylinder [1] are calculated by integrating the hydrostatic as well as hydrodynamic forces acting on the cylinder. The forces associated with the structure itself, i.e. the restoring, inertial and gravity forces acting on the structure, are taken into account in the kinetic and potential energies of the system. The hydrodynamic forces are incorporated partly in the kinetic energy and partly as generalized forces. In deriving the equations of motion, nonlinearities are introduced by Taylor expanding the trigonometric functions of the state variables.

The characteristic frequencies are determined by transforming the problem into an eigenvalue problem. As a control parameter (for example the fluid velocity) is varied, the modal frequencies evolve in the complex-frequency plane and, when the imaginary component of one of them crosses the imaginary axis, a bifurcation to flutter or divergence occurs. Once flutter develops, the amplitude grows with increasing flow velocity, until impacting with the outer flow-containing pipe occurs. This gives rise to a strongly nonlinear force, which in this paper is modelled either as a cubic spring (for analytical convenience) or, more realistically, by a trilinear spring model.

Solutions of the equations were obtained by using a fourth-order Runge-

Kutta integration algorithm, with a step size of 0.01. For the purpose of checking the convergence of the solution, the Runge-Kutta-Fehlberg algorithm was also used and same results were obtained. The results showed that, as the flow is incremented, the Hopf bifurcation is followed by a cascade of period-doubling bifurcations leading to chaos.

In numerical simulation, the following values of nondimensional parameters were used: mass $\beta=0.4$, gravity $\gamma=10$, length $\varepsilon=10$, base $C_b=0.1$ and friction coefficient $C_f=0.025$. The simulations were conducted by changing the drag coefficient ($C = 0.2, 0.63, 1$), the form coefficient ($f = 0, 0.2, 0.4, 0.6, 0.8$), as well as the number of degrees of freedom ($n = 2, 3$). For constant values of C and n, the critical velocities decrease with increasing f. For constant values of C and f, the critical velocities decrease with decreasing n.

The dynamical behaviour is illustrated by phase portraits (Fig.1) and bifurcation diagrams (Fig.2). The difference between the quasi-periodic and chaotic behaviour of the system is examined and characterized by employing the techniques of power spectra, Poincare maps and Lyapunov exponents.

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Fig.1: C=0.2, f=0.8, n=2, u=2.79

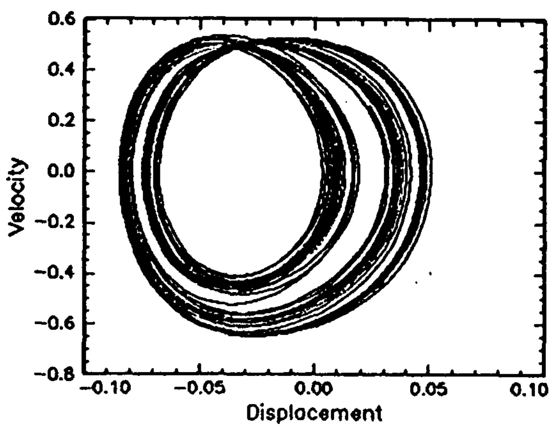
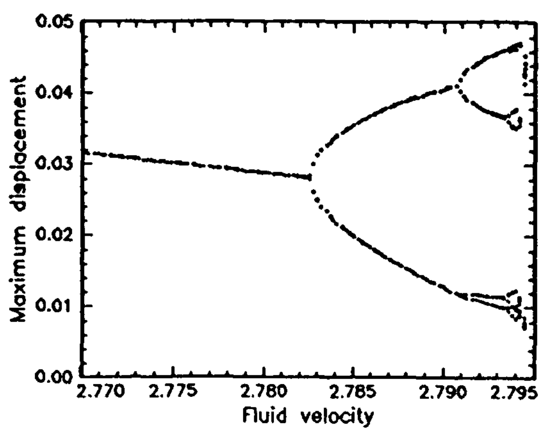


Fig.2: C=0.2, f=0.8, n=2



WEAK AND STRONG INTERACTIONS IN VORTEX-INDUCED RESONANT VIBRATIONS OF CYLINDRICAL STRUCTURES

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Abstract

Flow-induced vibrations of elastic structures are of great practical importance in engineering designs. For bluff elastic structures, such as most of the civil engineering structures, vortex shedding is the main fluid dynamic mechanism for exciting vibrations of the structures. Any structure with a sufficiently bluff trailing edge sheds vortices in subsonic flows in a very wide range of Reynolds number. Vortices are shedded alternately from each side of the structure and then generate an oscillating pressure field on the structure. This oscillating pressure field is the source of periodic forces which can cause the elastic structure to vibrate with a large amplitude and thus may lead to destructive effects on the structure.

In order to make a theoretical analysis on vortex-induced vibrations, wake oscillator models were proposed to predict the resonant response of the vortex-excited vibrations of circular cylinders normal to the flow. In these models, the dynamic interaction between the structure and the flow is described by nonlinear oscillator equations. The structure is modeled as a linear oscillator, and the vortex wake is described as a nonlinear, self-excited oscillator coupled to the structure. The model parameters can be determined by the properties of the fluid and the structure as well as experimental data. Although they are not the models obtained directly from the analysis of the fluid field and cannot give details about the fluid-structure interaction, the wake oscillator models do predict many of the dynamic effects that have been observed experimentally. For example, they are successfully used to estimate the maximum amplitude of response and exhibit the frequency entrainment phenomenon when the vortex shedding frequency approaches the natural frequency of the structure. However, in previous analyses of the wake oscillator models, only the case of main inner-resonance was considered. Effects of the intensity of fluid-structure interaction as well as the elastic nonlinearity of structures have not been investigated.

This investigation thus treats vortex-induced resonant vibrations of cylindrical structures with weak and strong fluid-structure interactions. Theoretical analyses for the subharmonic and superharmonic inner-resonances in the case of strong interaction , as well as the main inner-resonance in the case of weak interaction, are made by using the method of multiple scales. The effect of nonlinear elasticity is considered. Both the theoretical and numerical results obtained in the present investigation show that much more complicated dynamical phenomena, including large amplitude subharmonic, superharmonic, quasiperiodic and chaotic vibrations, can take place when the fluid-structure interaction is strong. Theoretical predictions are verified by numerical integration results using the fourth-order Runge-Kutta algorithm. The maximum amplitude of response in the case of weak interaction obtained by using the formula of this paper is also in good agreement with the corresponding experimental and theoretical results available in the literature.

It is concluded that the method of multiple scales gives excellent theoretical results of subharmonic and superharmonic inner-resonances in the case of strong fluid-structure interaction, and main inner resonances in the case of weak fluid-structure interaction. The method of multiple scales with first order uniform expression of the solution , however , is not applicable to the study of main inner-resonance for strong fluid-structure interaction.

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AN EFFICIENT NUMERICAL TECHNIQUE FOR THE ANALYSIS OF PARAMETRICALLY EXCITED NONLINEAR SYSTEMS

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ABSTRACT

Nonlinear ordinary differential equations with periodic coefficients occur in many practical applications like stability of structures under periodic loading or boundary conditions, helicopter rotor dynamics in forward flight, dynamic analysis of a satellite with an eccentricity in its orbit etc. Nonlinear periodic systems generally exhibit rich dynamic behavior characterized by periodic, period multiplication, and chaotic responses. Small changes in the parameter values or the initial conditions could lead to entirely different types of dynamic behavior. The computational scheme used for such dynamical systems should therefore be able to predict the type of response accurately for small changes in the parameter values or initial conditions.

Some of the popular numerical schemes for nonlinear initial value problems include Runge-Kutta and Newmark methods. The accuracy of these methods are known [1] to depend strongly on the step size. The choice of an optimal step size could lead to significant computational time [2]. The use of weighted residual methods (finite elements in time) leads [3] to a set of nonlinear algebraic equations in the unknown variables, the solution of which is generally expensive.

A new computational scheme is proposed in the following which has many attractive features over the ones mentioned above. This method is an extension of the superefficient scheme proposed earlier [4], to nonlinear problems. It is illustrated that the present technique is an efficient tool for computing periodic, multi-periodic and chaotic solutions.

<u>PROPOSED SCHEME</u>: The object for the proposed scheme is a set of second-order nonlinear ordinary differential equations with periodic coefficients given by

$$\ddot{\boldsymbol{y}}(t) + [\boldsymbol{V} + \boldsymbol{V}^*(t)]\dot{\boldsymbol{y}}(t) + [\boldsymbol{U} + \boldsymbol{U}^*(t)]\boldsymbol{y}(t) = \boldsymbol{F}(\boldsymbol{y}, t) \tag{1}$$

with appropriate initial conditions $\boldsymbol{y}(0)$ *and* $\dot{\boldsymbol{y}}(0)$. **y**(t) is an nx1 state vector, **F(y,t)** is an nx1 periodic vector of nonlinear elements of the state variables. $\boldsymbol{U}^*(t)$ *and* $\boldsymbol{V}^*(t)$ are periodic square matrices. **U** and **V** are constant square matrices.

The computational scheme follows that of a Picard type iteration given by

$$\ddot{\boldsymbol{y}}_{i+1}(t) + [\boldsymbol{V} + \boldsymbol{V}^*(t)]\dot{\boldsymbol{y}}_{i+1}(t) + [\boldsymbol{U} + \boldsymbol{U}^*(t)]\boldsymbol{y}_{i+1}(t) = \boldsymbol{F}(\boldsymbol{y}_i, t) \qquad i = 0,1,2,\ldots \tag{2}$$

The iteration is started with an initial guess of $\mathbf{y}_0=0$. Equation (2) implies the method requires the solution of only a set of linear equations with periodic coefficients in each iteration. The method proposed in reference [4] is adopted for the solution of the set of linear equations (2). The periodic square matrices $\boldsymbol{U}(t)$ *and* $\boldsymbol{V}(t)$ and the known vector $\mathbf{F}(\mathbf{y}_i,t)$ are expanded in Chebyshev polynomials with known coefficients and the state vector $\mathbf{y}_{i+1}$ is expanded in Chebyshev polynomials with unknown coefficients $\boldsymbol{B}'_{i+1}$ in conjunction with the integrations of equations (2). Convergence is guaranteed when the norm given by $\boldsymbol{B}'_{i+1} - \boldsymbol{B}'_{i} \le \epsilon$ is satisfied. ϵ is the predetermined error constant.

<u>NUMERICAL RESULTS AND CONCLUDING REMARKS</u>: The computational scheme described above is applied to the nonlinear equation with periodic coefficient given by

$$\ddot{u}(t)+2c\dot{u}(t)+(\omega_0^2+P\cos\Omega t)u(t)+\alpha u^2(t)+\beta u^3(t) = 0 \qquad (3)$$

The above equation was previously studied by Stupnicka et al [5] and was shown to exhibit periodic, period doubling and chaotic responses for the following values of the parameters

$$c=0.1,\ \omega_0=1,\ P=0.9,\ \alpha=1.5,\ \beta=0.5,\ u(0)=0.02 \ \textit{and}\ \dot{u}(0)=0 \qquad (4)$$

for various exciting frequencies. They have used conventional numerical integration (Runge-Kutta) to compute the various responses.

It is observed in figures 1 to 4 that the present method is as accurate as the Runge-Kutta (R-K) method in obtaining periodic, period doubling and chaotic responses for small variations in the exciting frequency. The present method requires about 6-8 term Chebyshev expansion in each period $(2\pi/\Omega)$ of integration for convergence in less than ten iterations in all the cases considered.

In conclusion, it is interesting to note the following merits of the present scheme over the conventional methods:

1. The CPU time taken by the present method is about half of that taken by the R-K method for a given response.
2. Since the present method requires only matrix inversion and multiplications, it enables parallel computations making it attractive for the analysis of large scale systems.
3. Unlike the harmonic balance and the weighted residual methods, the present approach does not require the solution of a set of nonlinear algebraic equations.
4. Last but not the least, the proposed scheme does not require the equations of motion to be rewritten in the state-space form.

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A Mesh Repartitioning Scheme to Cope with Nonlinearities Resulting from Large and Fast Rotations of Deformable Bodies

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Abstract

It has been shown by various researchers that, for a deformable body undergoing large and fast rotation, linear finite-element approaches will produce erroneous results if the nonlinear coupling between the deformation and rigid-body coordinates are not accounted for properly in the equations of motion. Furthermore, if the deformation of any given single finite element is such that the usual geometric approximations cannot be made additional serious errors may result. Conversely, when provisions are made in dynamic formulations to account for these nonlinear effects, the computational cost becomes prohibitive because of the very large number of additional terms involved. The scheme proposed in this paper relies on the observation that, for a general deformable body, only some portions of the body undergo deformation which is strongly coupled with large and high rates of rotation or which render geometric approximations invalid. As a result, the dynamics of most of the body can be treated using linear finite elements, and provisions have to be made to account for nonlinear effects for only some of the elements. However, because the location and the number of such elements depend on the instantaneous dynamics of the body and change over time as the body moves through space, it is impossible to decide on an appropriate finite-element mesh for the body.

This paper describes a method whereby such elements are identified at selected time steps and the finite-element mesh describing the deformation of the body is reconstituted appropriately to account for nonlinearities due to the coupling between rigid and deformation coordinates and geometric nonlinearities due to large-angle deformations.

A very flexible beam undergoing large and fast rotation but small strain is used as an example. The results of the approach are compared to standard finite-element results and fixed-mesh results with nonlinear analysis. Computational loads and prospects for implementation on parallel processors are also discussed.

RESPONSE OF MULTI-DEGREE-OF-FREEDOM SYSTEMS WITH GEOMETRICAL NONLINEARITY UNDER RANDOM EXCITATIONS BY THE STOCHASTIC CENTRAL DIFFERENCE METHOD

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Abstract

There are various simple direct integration schemes for random response of linear and nonlinear mechanical and structural systems in the literature. With linear or linearized multi-degree-of-freedom (MDOF) systems the method of DiPaola and associates requires the classical modal transformation, while that of Hoshiya and associates makes use of the white noise characteristics of the autoregressive (AR) model in addition to the normal modal analysis. Both methods are essentially of the Euler algorithm type and hence have much larger truncation errors than that applying the stochastic central difference (SCD) method and various other stochastic direct integration schemes developed by To. The SCD method is a simple and versatile numerical integration algorithm that has proved to be very accurate and efficient for linear and nonlinear MDOF systems under stationary and nonstationary random excitations.

In the present work the SCD method is extended to the determination of response statistics of MDOF systems with geometrical nonlinearity. Examples are plate and shell structures, represented as MDOF systems by the finite element method (FEM) or boundary element method (BEM) , subjected to transversal and in-plane random forces. Results for a simple two degree-of-freedom (DOF) system under nonstationary random excitations are compared with those obtained by another tested method employing Ito's calculus. A procedure is also presented to circumvent the numerical instability occurred in stiff structures and systems with high natural frequencies. Thus, the proposed technique and its deterministic counterpart can easily and efficiently be applied to response analysis of linear and nonlinear MDOF systems involving wave propapation and impact. The proposed technique is also simple to use and accurate.

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